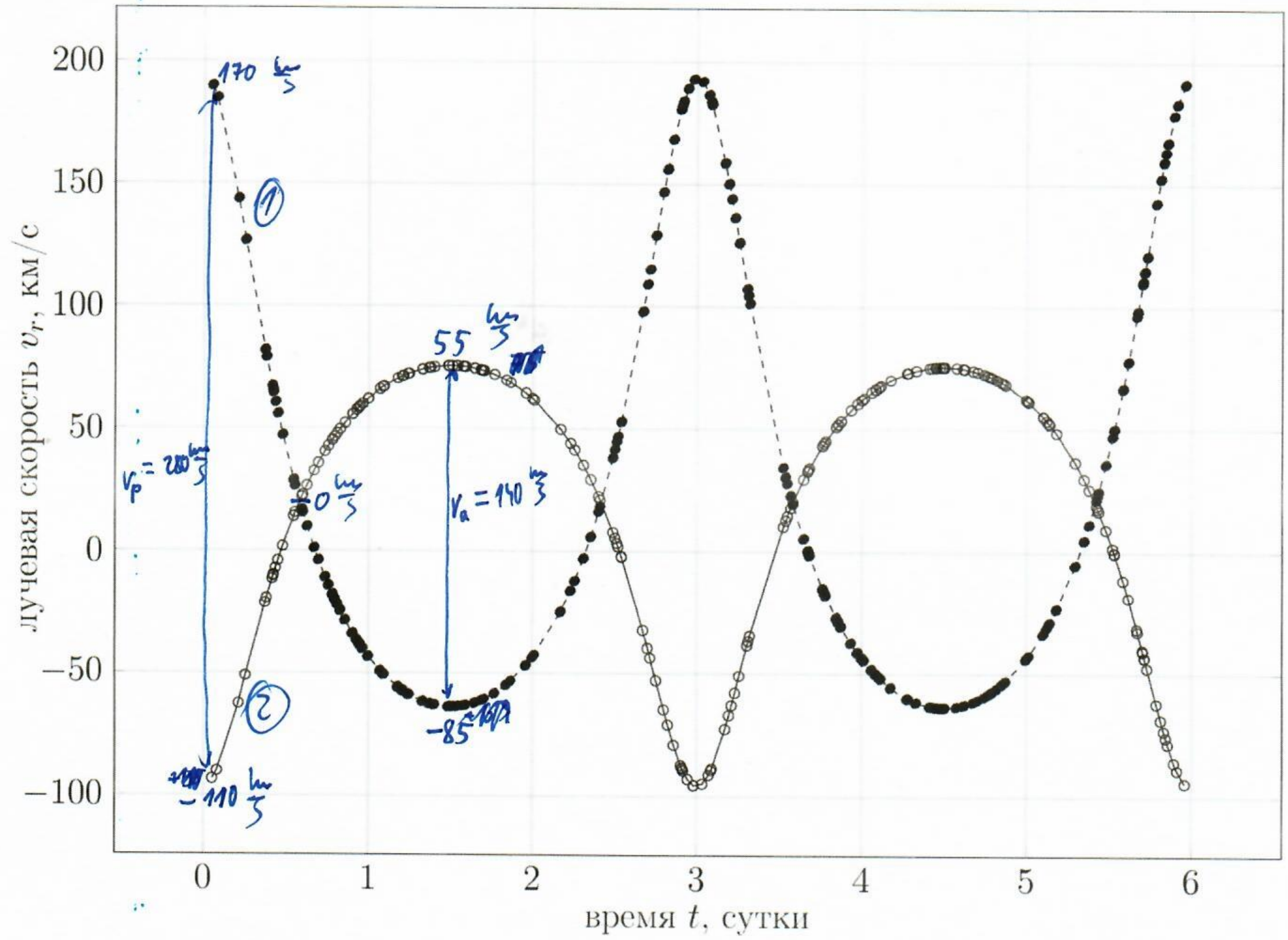
11 класс

You are given a radial velocity curve for a binary system consisting of two Main Sequence stars. The line of sight lies in the plane of the orbit, the line of apsides (connecting the periastrons and apoasters of the orbits) is perpendicular to the line of sight. Find the parameters of the system: the masses of the stars, the period and semi-major axis of the system, the eccentricity of the orbit. Determine the apparent magnitudes of the system at maximum and minimum brightness. The annual parallax of the system is $\pi = 0''.05$, the stars are assumed to be spherically symmetric, and the effects of heating and darkening of the disk towards the edge can be neglected.



For sake of clarity and conciseness, I have written the numerical calculations to the end.

At first, note that the system has a systemic velocity. If we are to ~~that~~ properly analyse the properties, we should ensure that we are working in the barycentre reference frame.
The origin of the graph on page 1 is then the intersection of the given lines. The relative speeds ~~without (the unit)~~ are calculated on the graph using Galilean transformation.
~~The~~ ~~su~~ Let's apply one more transformation: this time to the star 2 (white circles) as an origin.
The associated ~~m~~ absolute values of the extreme velocities (periapsid and apoapsid, respectively) are:

$v_p = 280\ \frac{km}{s}$ $\quad v_a = 140\ \frac{km}{s}$

Using vis-viva equation:

$$v_p^2 = GM\left(\frac{2}{r}-\frac{1}{a}\right) = \frac{GM}{a}\left(\frac{2}{1-e}-1\right) = \boxed{\frac{GM}{a}\frac{1+e}{1-e} = v_p^2} \leftarrow \underline{M = m_1 + m_2}$$

Similalilly,

$$\boxed{v_a^2 = \frac{GM}{a}\frac{1-e}{1+e}}$$

Combining them gives

$$(v_a v_p)^2 = \left(\frac{GM}{a}\right)^2 \rightarrow v_a v_p = \frac{GM}{a}$$

Applying Kepler's 3rd law:

$$\frac{a^3}{t_0^2} = \frac{GM}{4\pi^2}$$

we obtain

$$a^3 = \frac{GM t_0^2}{4\pi^2} = \frac{G^3M^3}{v_a^3 v_p^3} \quad \text{and} \quad \boxed{M = \frac{t_0 v_p v_a}{2\pi G}\sqrt{v_p v_a}}$$

Using $\boxed{\underline{t_0 = 3{,}0\,d}} = 3{,}0 \cdot 86400\ s$, the mass of the system is,

$M = \boxed{5{,}2 \cdot 10^{30}\ kg} = m_1 + m_2$ (For numerical calculations, ~~...~~ see the Appendix chapter)

Returning to the barycentre reference frame, apply the identity for the centre of mass:

$m_1 \vec{r}_1 + m_2 \vec{r}_2 = 0 \rightarrow$ differentiate $\rightarrow m_1 \dot{\vec{r}}_1 + m_2 \dot{\vec{r}}_2 = 0$

Therefore, with taking absolute values,

$$\frac{m_1}{m_2} = \frac{v_2}{v_1}$$

Specifically, consider $v_1 = v_{1a}$, $v_2 = v_{2a}$ (at apoapsid), and note that $M = m_1 + m_2$

$$M - m_2 = \frac{v_2}{v_1} m_2 \rightarrow m_2 = \frac{M}{\left(\frac{v_2}{v_1}+1\right)} = \frac{M v_1}{v_2 + v_1} = \boxed{3{,}2 \cdot 10^{30}\ kg}$$

(again, every numerical calculation is in the Appendix chapter)

~~Similarly~~ ~~Similarly m_1~~ ~~M~~ Similarly $m_1 = M - m_2 = \boxed{2{,}0 \cdot 10^{30}\ kg}$

Returning to the identity at the beginning of the paper,

$$a = \frac{GM}{v_p v_a} = a_1 + a_2 = \boxed{8,7 \cdot 10^{15}\ m}$$

Also,

$m_1 a_1 = m_2 a_2$

Solving the system of 2 equations,

$$m_1 a - m_1 a_2 = m_2 a_2$$

$$a_2 = \frac{m_1 a}{M} = \boxed{3,4 \cdot 10^{15}\ m}$$

$$a_1 = a - a_2 = \boxed{5,3 \cdot 10^{15}\ m}$$

Returning to the 2nd body reference frame and the equation from the beginning:

$$\frac{GM}{a}\frac{1+e}{1-e} = v_p^2 \rightarrow \eta = \frac{v_p^2 a}{GM} = \frac{v_p^2}{GM}\frac{GM}{v_p v_a} = \frac{v_p}{v_a} = 2$$

$$\frac{1+e}{1-e} = \eta$$

$$1 + e = \eta - e\eta$$

$$e(1+\eta) = \eta - 1$$

$$e = \frac{\eta - 1}{\eta + 1} = \boxed{0,33}$$

$m_1 = 1,0\ M_\odot$

$m_2 = 1,6\ M_\odot$

$d = \frac{1}{\pi} = ~~24~~\ 20\,pc$

Using the relation

$$L \propto M^{3,5} \rightarrow L = M^{3,5}$$

$[M_\odot]$, $[L_\odot]$

$L_1 = 1,0\ L_\odot$

$L_2 \doteq 5,6\ L_\odot$

Maximum brightness: both stars are visible.

$L_{TOT} = 6,6\,L_\odot$

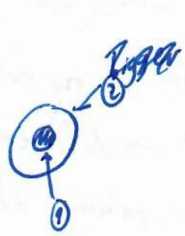
$$M_2 - M_1 = -2,5 \log \frac{j_2}{j_1}$$

$$M_2 = -2,5\log(6,6) + 4,8 = 2,8$$

therefore $m - M = 1,5$

$$m = M + 5\log\left(\frac{d}{10pc}\right) = 2,8 + 5\log(2) = \boxed{4,3}$$

Minimum magnitude:



Because the radius of the main branch star is approximately

$R \propto M$

it follows

$R_1 = 1\ R_\odot$

$R_2 = 1,6\ R_\odot$

Using that we can find the effective luminosity at the time of an eclipse:

very easy calculation because of $R_\odot$ / ~~$M_\odot$~~ / $L_\odot$ units

$$L_{eff} = \frac{(\cancel{\pi} R_2^2 - \cancel{\pi} R_1^2) L_2 + \cancel{\pi} R_1^2 L_1}{\cancel{\pi} R_2^2} = \frac{R_2^2 L_2 - R_1^2 L_2 + R_1^2 L_1}{R_2^2} = 4{,}0\, L_\odot$$

Again, the calculation for visual magnitude:

$$M_2 = -2{,}5 \log(4) + 4{,}8 = 3{,}3$$

~~Modulus distan~~

Distance modulus is the same $(m-M)$, so

$$m = M + 1{,}5 = \boxed{4{,}8}$$

All in all:

Star 1:
$m_1 = 2{,}0 \cdot 10^{30}$ kg
$a_1 = 5{,}3 \cdot 10^{15}$ m

Star 2:
$m_2 = 3{,}2 \cdot 10^{30}$ kg
$a_2 = 3{,}4 \cdot 10^{15}$ m

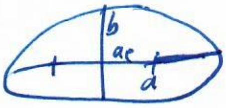
System:
$a = 8{,}7 \cdot 10^{15}$ m
$e = 0{,}33$
$t_0 = 3{,}0$ d
$M = 5{,}2 \cdot 10^{30}$ kg

Magnitudes:
$m_{MIN} = 4{,}3$
$m_{MAX} = 4{,}8$

Appendix

$a_2(1-e_2)+a_1(1-e_1)=a(1-e)$

$a=a_1+a_2$



$2a=2\sqrt{a^2e^2+b^2}$

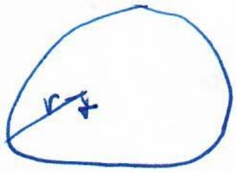
$a^2=a^2e^2+b^2$

$\frac{a^2-b^2}{a^2}=e^2$

$\sqrt{1-\frac{b^2}{a^2}}$

$r=a(1-e)$

$2(a_1+a_2)-\cancel{a(1-e)}=2a(1+\cancel{e})+a(\cancel{1+e}-\cancel{e})$



$v^2=GM\left(\frac{2}{r}-\frac{1}{a}\right)=\frac{GM}{a}\left(\frac{2}{1-e}-1\right)=\frac{GM}{a}\frac{1+e}{1-e}=v_p^2$

$\frac{GM}{a}\frac{1-e}{1+e}=v_a^2$

$v_pv_a=\frac{GM}{a}$

$\frac{a^3}{t_0^2}=\frac{GM}{4\pi^2}$

$a=\sqrt[3]{\frac{GMt_0^2}{4\pi^2}}=\frac{GM}{v_pv_a}$

$\sqrt[3]{\frac{Gt_0^2}{4\pi^2}}=\frac{G^3M^2}{v_p^3v_a^3}$

$M=\sqrt{\frac{t_0^2v_p^3v_a^3}{4\pi^2G^2}}$

$\frac{\frac{1+e}{1-e}}{\frac{1-e}{1+e}}=\left(\frac{1+e}{1-e}\right)^2=\left(\frac{v_p}{v_a}\right)^2$

$\frac{1+e}{1}$

$M=\frac{t_0v_pv_a}{2\pi G}\sqrt{v_pv_a}=\frac{3\cdot 86400\cdot 140\cdot 280\cdot 10^6}{2\cdot 3{,}14\cdot 6{,}67\cdot 10^{-11}}\sqrt{140\cdot 280}\cdot 10^{3}\ \mathrm{kg}=\frac{8{,}64\cdot 1{,}4\cdot 2{,}8\cdot 1{,}4}{2\cdot 6{,}67}\cdot 10^4\cdot 10^2\cdot 10^2\cdot 10^2\cdot 10^{20}\ \mathrm{kg}=$

$=\frac{8{,}63\cdot 9{,}4}{6{,}67}\cdot 10^{30}\ \mathrm{kg}=\frac{34{,}56}{6{,}67}\cdot 10^{30}\ \mathrm{kg}=\boxed{5{,}2\cdot 10^{30}\ \mathrm{kg}}$

$\boxed{t_0=3{,}0\ d}$

$\sqrt{140\cdot 280}=\sqrt{2}\cdot 140$

$3{,}4\cdot 3{,}4=9{,}96$

$\frac{8{,}64\cdot 4}{34{,}56}$

$5/8 \to 5$

$m_1+m_2=M$

$m_1a_1=m_2a_2$

$a=a_1+a_2$

$Ma_1-m_2a_1=m_2a_2$

$=\frac{m_1a_1}{(m_1+m_2)a_1}$

$\frac{Mm_2a_2}{m_1}-\frac{m_2^2a_2}{m_1}=m_2a_2$

$m_1\vec{v}_1+m_2\vec{v}_2=0$

$m_1\vec{v}_1+m_2\vec{v}_2=0$

$m_1\vec{v}_1=-m_2\vec{v}_2$

$m_2=+\frac{m_1v_1}{v_2}$

$m_1r_1=m_2r_2$

$m_1\dot{r}_1=m_2\dot{r}_2$

$\frac{m_1}{m_2}=\frac{v_2}{v_1}$

$m_1=M-m_2$

$M-m_2=\frac{v_2}{v_1}m_2$

$m_2\left(\frac{v_2}{v_1}+1\right)=M$

$m_2=\frac{Mv_1}{v_2+v_1}=5{,}2\cdot 10^{30}\ \mathrm{kg}$

$\frac{140}{280}\cdot 10=\boxed{3{,}2\cdot 10^{30}\ \mathrm{kg}}$

$m_1=\boxed{2{,}0\cdot 10^{30}\ \mathrm{kg}}$

$m_1a-m_1a_2=m_2a_2$

$a_2=\frac{m_1a}{m_2+m_1}=\frac{m_1a}{M}=$

$=\frac{2{,}1}{5{,}2}$

6/6

$a = \frac{GM}{v_p v_a} = \frac{6{,}67\cdot10^{-11}\cdot5{,}2\cdot10^{30}}{2{,}8\cdot1{,}4\cdot10^{4}}\,m = \frac{6{,}7\cdot5{,}2}{4}\cdot10^{15}\,m = 8{,}7\cdot10^{15}\,m$

$a_2 = \frac{8{,}8}{5{,}2}\cdot10^{15}\,m = 3{,}4\cdot10^{15}\,m$

$e = \frac{v_p^2 a - GM}{v_p^2 a + GM} = 2{,}8\cdot8{,}7\cdot10^{25} - 6{,}67\cdot10^{-11}\cdot5{,}2\cdot10^{30}$

$\frac{2{,}8\cdot8{,}7\cdot10^{-8}\cdot10^{11}}{6{,}67\cdot5{,}2} = \frac{2{,}8\cdot8{,}7}{6{,}67\cdot5{,}2}\cdot10^{6} =$

$\frac{v_p - v_a}{v_p + v_a}$ $\quad \frac{28-14}{28+14} =$

$d = \frac{1}{\pi} = \frac{1}{\frac{1}{20}} = 20\,pc$

$\frac{1}{3}$

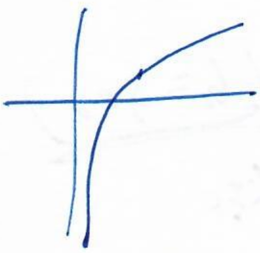
$1{,}6^{3{,}5} \rightarrow$

$1{,}6 \rightarrow 2{,}6 \rightarrow 4{,}1 \rightarrow 6{,}6$ $\quad 5{,}7 \rightarrow 5{,}6$

$\log 6{,}6 \approx 0{,}8$

$-\frac{8}{2}\cdot\frac{8}{10} + \frac{48}{10}$

$-2 + 4{,}8$



$\approx 0{,}3$

$4{,}8 + 1{,}5$

$L_{eff} = \frac{(\pi R_2^2 - \pi R_1^2)L_2 + \pi R_1^2 L_1}{\pi R_2^2} =$

$= \pi(R_2^2 L_2 - R_1^2 L_2 + R_1^2 L_1) =$

$= \frac{\pi(1{,}6^2\cdot5{,}6 - 1\cdot5{,}6 + 1\cdot1)}{\pi\,1{,}6^2} =$

$= \frac{14{,}6 - 5{,}6 + 1}{2{,}6} = \frac{10}{2{,}6} = 4$

$100:2$

$-2{,}5\cdot0,$

$-\frac{8}{2}\cdot\frac{6}{10} + 4{,}8$