

XXIX Санкт-петербургска олимпиада
по астрономия

Теоретичен тур

6 февруари 2022.

①

$g_{\oplus} = \frac{GM_{\oplus}}{R_{\oplus}^2}$ - земно ускорение

на полюса -
без центробежна
сила

$g = \frac{GM}{R^2}$ - ускорение на повърхността на
голямата планета

$R_{\oplus} \approx 6000 \text{ km} \Rightarrow R = 60000 \text{ km} = 10 R_{\oplus}$

$g_{\oplus} = g \Rightarrow \frac{GM_{\oplus}}{R_{\oplus}^2} = \frac{GM}{(10R_{\oplus})^2}$

$\frac{M_{\oplus}}{R_{\oplus}^2} = \frac{M}{100R_{\oplus}^2}$

$\Rightarrow M = 100 M_{\oplus} = 6 \cdot 10^{26} \text{ kg}$

от III Закон на Кеплер

$\frac{a^3}{T^2} = \frac{GM}{4\pi^2}$

$\frac{a_c^3}{T_c^2} = \frac{GM_{\oplus}}{4\pi^2}$

$\Rightarrow \frac{a[a_c]}{T[T_c]^2} = M[M_{\oplus}]$

$\Rightarrow a = \sqrt[3]{100} \cdot a_c = 4,65 \cdot 384400 \approx$

$\approx 1\,790\,000 \text{ km}$

радиус на
орбитата на
малката планета

$\frac{2R_c}{a_c - R_{\oplus}}$

$= \frac{2R_1}{a - R}$

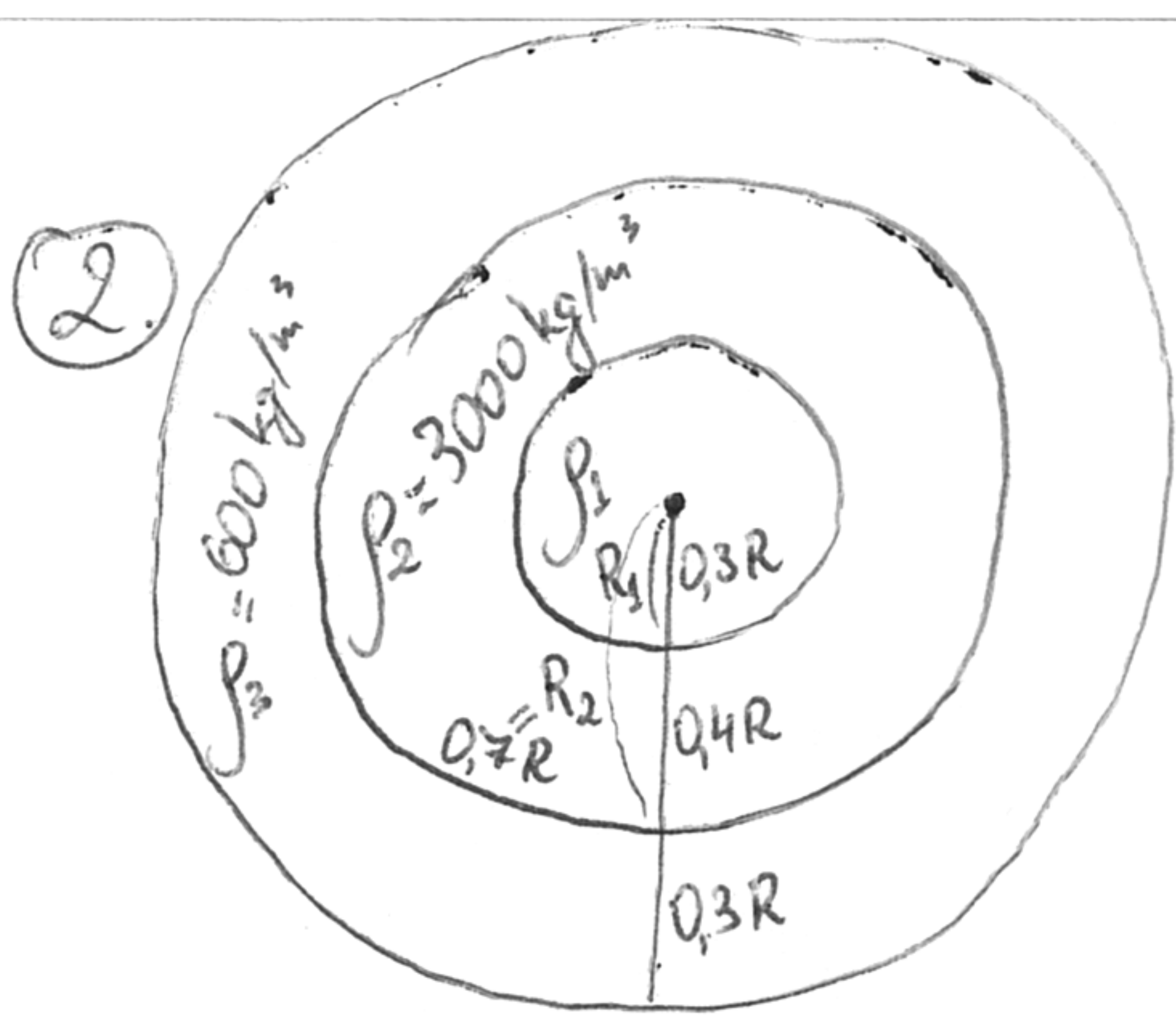
радиус на
малката
планета

$\Rightarrow \frac{R_c}{a_c - R_{\oplus}} = \frac{R_1}{a - R}$

$\frac{1700}{380000} = \frac{R_1}{1730000}$

$384400 - 6400 = 378000 \approx 380000$

$\Rightarrow R_1 \approx 7700 \text{ km}$



R - целият радиус

$\rho = 1530 \text{ kg/m}^3$ - средна плътност

$$M = \frac{4}{3} \pi R^3 \rho$$

обем на сферичката

$$M = \frac{4}{3} \pi R_1^3 \rho_1 + \left(\frac{4}{3} \pi R_2^3 - \frac{4}{3} \pi R_1^3 \right) \rho_2 + \left(\frac{4}{3} \pi R^3 - \frac{4}{3} \pi R_2^3 \right) \rho_3 =$$

$$= \frac{4}{3} \pi (0,3R)^3 \rho_1 + \left(\frac{4}{3} \pi (0,7R)^3 - \frac{4}{3} \pi (0,3R)^3 \right) \rho_2 + \left(\frac{4}{3} \pi R^3 - \frac{4}{3} \pi (0,7R)^3 \right) \rho_3 =$$

$$= \frac{4}{3} \pi R^3 \left(0,3^3 \rho_1 + (0,7^3 - 0,3^3) \rho_2 + (1 - 0,7^3) \rho_3 \right) = \frac{4}{3} \pi R^3 \rho$$

$$\Rightarrow 0,027 \rho_1 + (0,343 - 0,027) \rho_2 + (1 - 0,343) \rho_3 = \rho$$

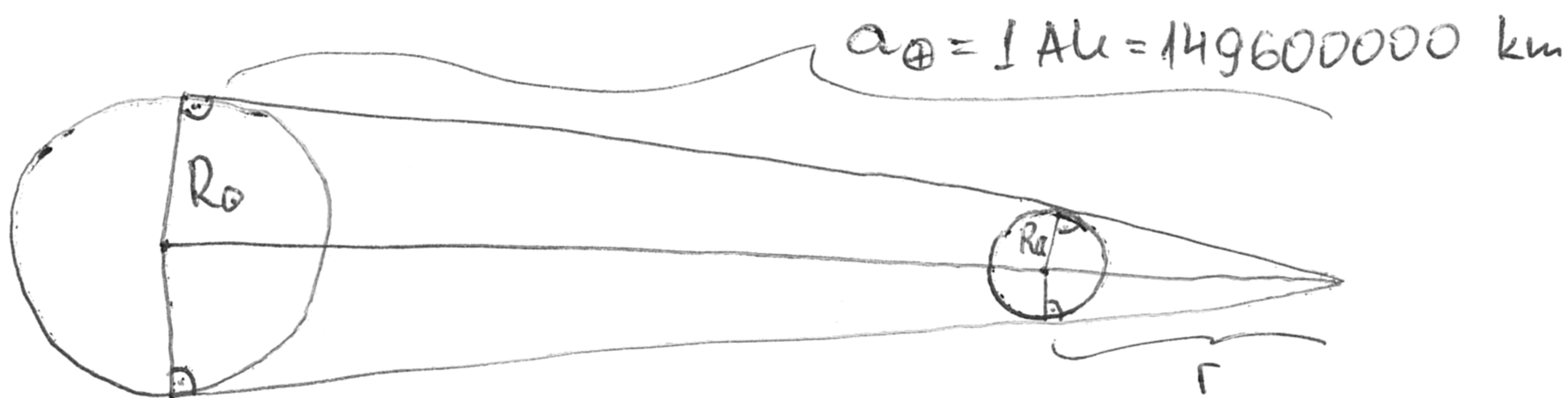
$$0,027 \rho_1 + 0,316 \cdot 3000 + 0,657 \cdot 600 = 1530$$

$$0,027 \rho_1 + 948 + 394,2 = 1530$$

$$0,027 \rho_1 + 1342,2 = 1530$$

$$\rho_1 = \frac{187,8}{0,027} = \frac{187800}{27} \approx \underline{\underline{6960 \text{ kg/m}^3}}$$

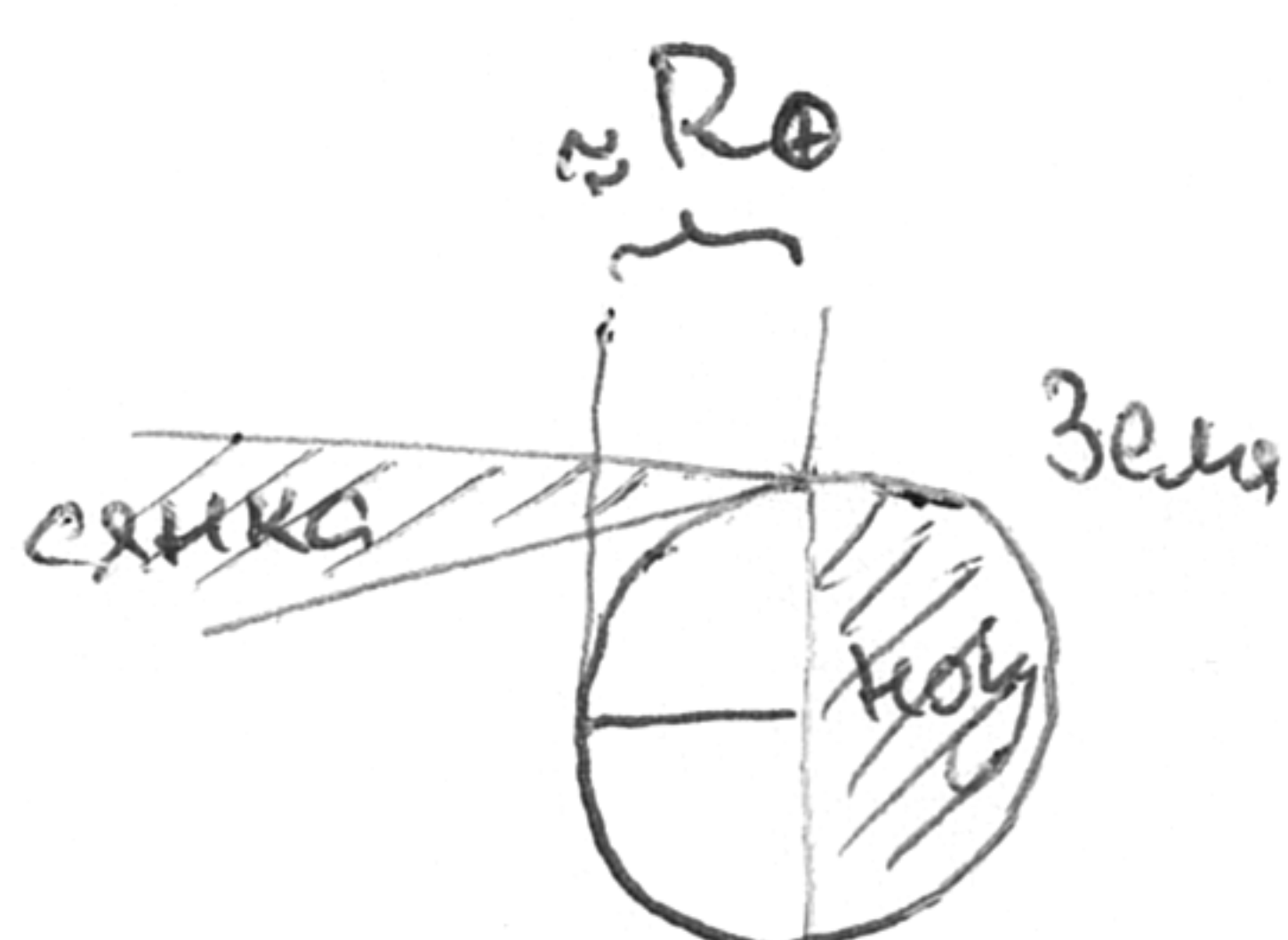
3.



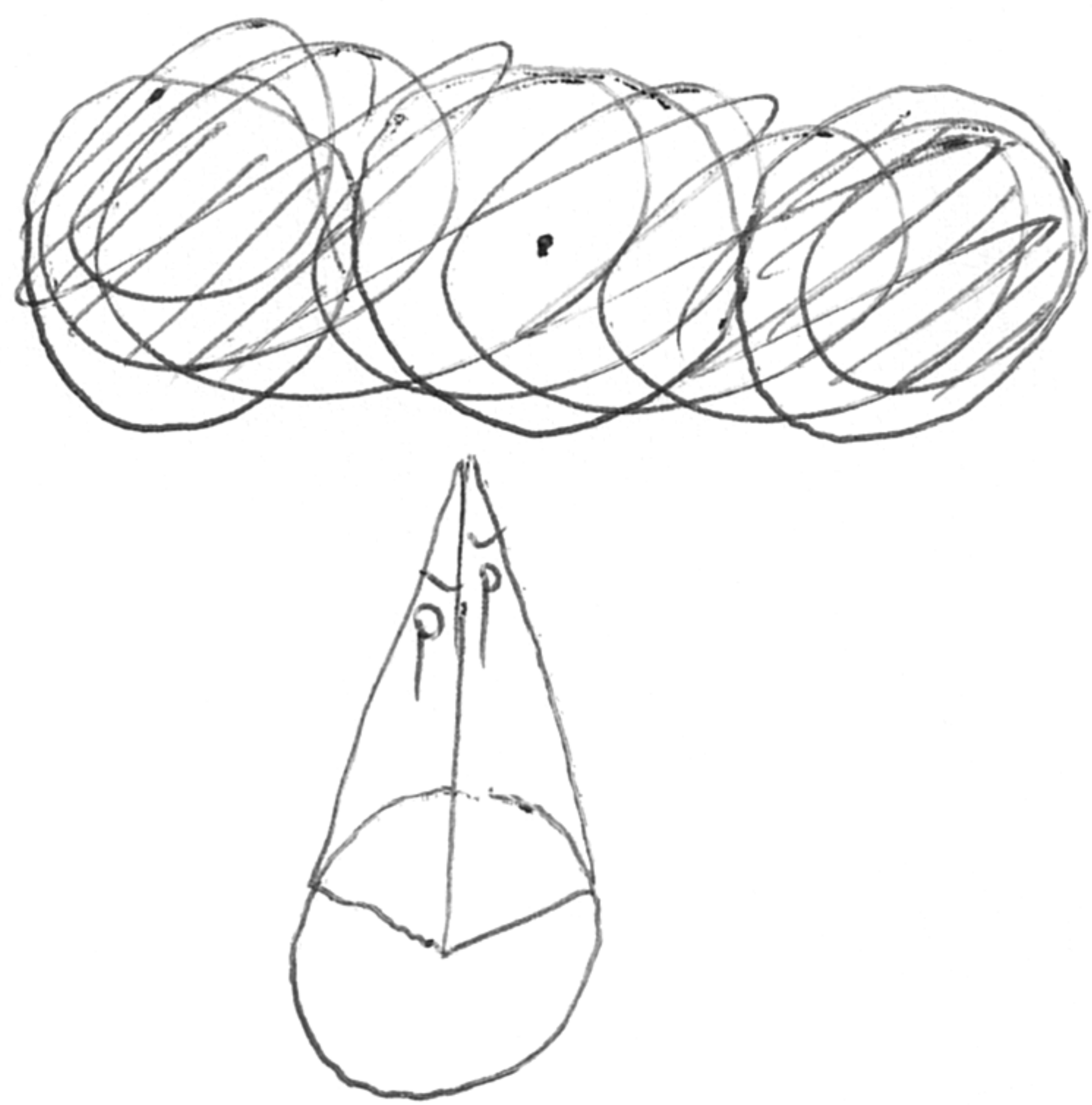
$$\frac{r}{R_{\oplus}} = \frac{a_{\oplus}}{R_{\oplus}}$$

$$\frac{r}{1737} = \frac{149600000}{696000} \approx \frac{150000}{700}$$

$\Rightarrow r \approx 372000$ km - между центровете на Земята и Луната



За да е повече време затъмнението, трябва в някой момент центровете на Слънцето, Луната и Земята да са на една права.



декоинжен хоризонтален паралел на Луната $\approx 1^\circ$

$$\frac{R_{\oplus}}{r} = \frac{R_{\oplus}}{372000} = \frac{6400}{200000} \approx 0.032$$

\Rightarrow Луната трябва да измине $2r$ за времето, в което някъде има тъмно слънчево затъмнен

$$\Rightarrow \frac{2,5^\circ}{360^\circ} = \frac{t}{29,5d}$$

$$t = \frac{2,5 \cdot 29,5}{360} \cdot 24 \approx 5h$$

половината от всички рожки са помитани

$$\frac{t}{T_{(y)}}$$

$$= \frac{5}{365,242} \cdot \frac{1}{2} = \frac{10000}{219}$$

$$\approx \frac{10000000}{220} \approx 50000 \text{ мол}$$

$$④ \quad R_1 \sim \sqrt[5]{E_1} \cdot t^{2/5}$$

$$R_2 \sim \sqrt[5]{E_2} \cdot t^{2/5} = \sqrt[5]{32E_1} \cdot t^{2/5} = 2\sqrt[5]{E_1} \cdot t^{2/5}$$

↳ по-мощната
свръхнова
⇒ $E_2 = 32E_1$

когато се срещат: $R_1 + R_2 = 300$ pc

$$\frac{R_1}{R_2} = \frac{\sqrt[5]{E_1} \cdot t^{2/5}}{2\sqrt[5]{E_1} \cdot t^{2/5}} = \frac{1}{2}$$

→ еднакво време от едновременното избухване до срещата на фронтите

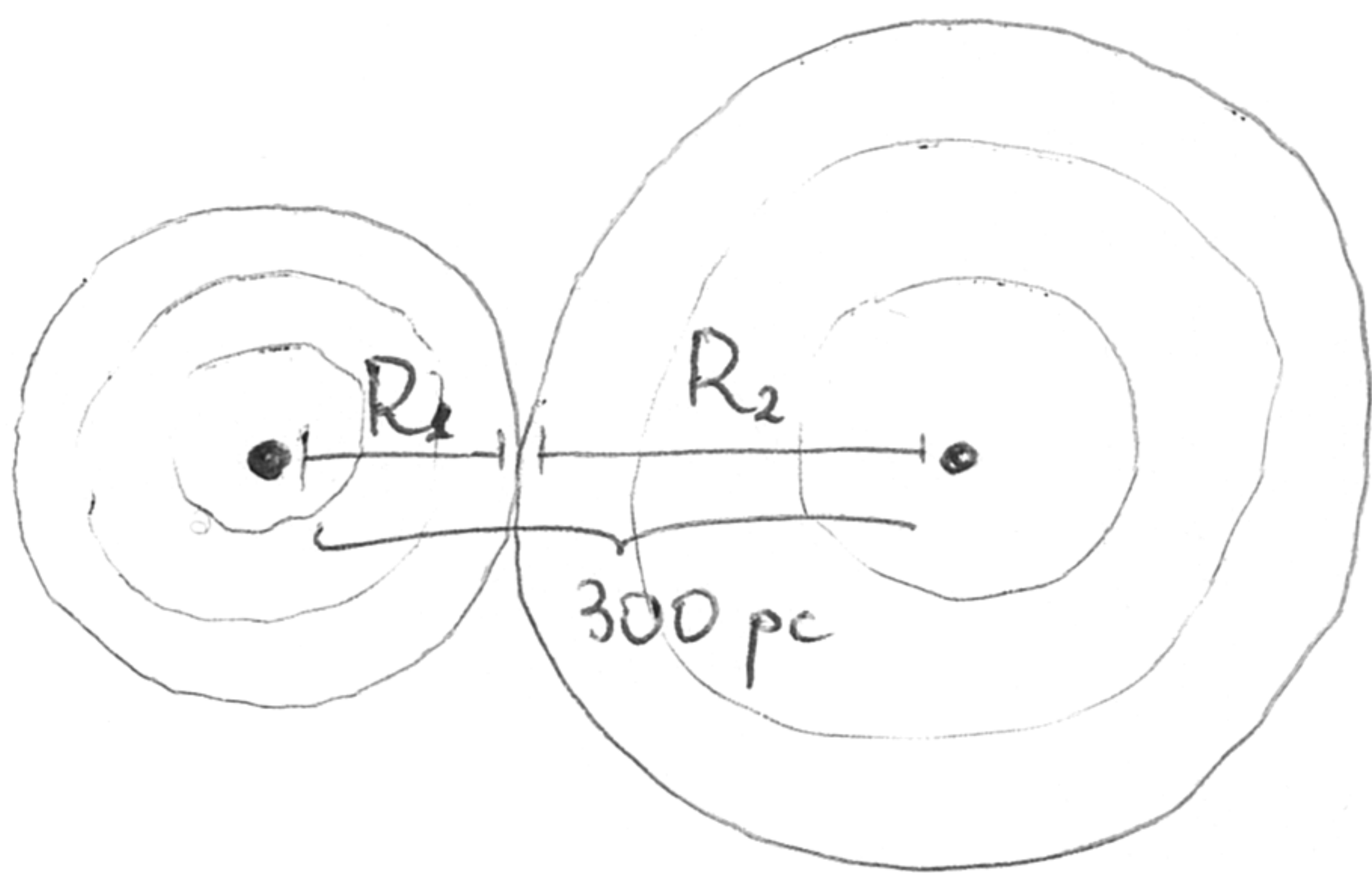
$$\Rightarrow R_2 = 2R_1$$

$$3R_1 = 300 \text{ pc}$$

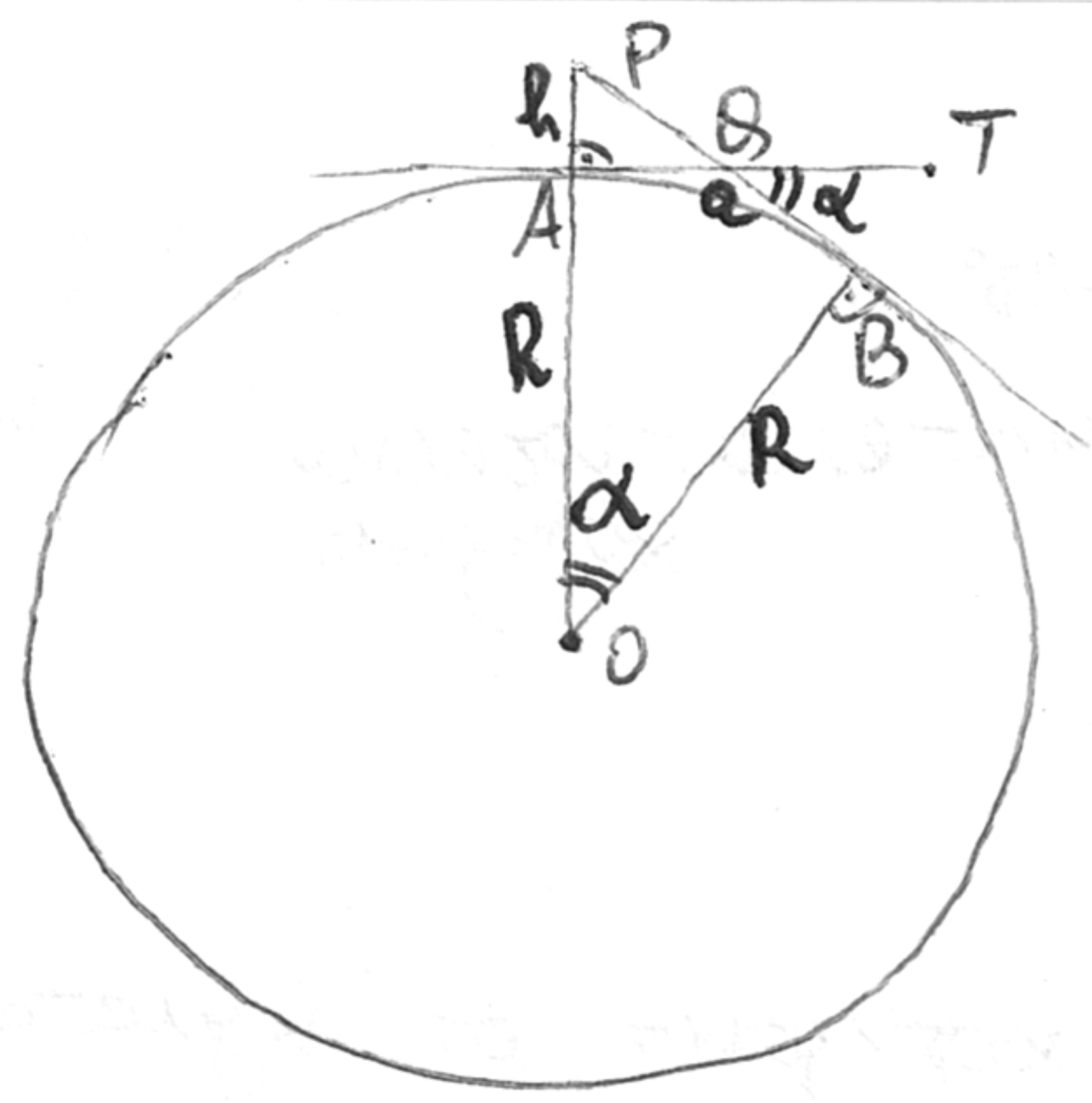
$$R_1 = 100 \text{ pc}$$

$$R_2 = \underline{\underline{200 \text{ pc}}} - \text{това е}$$

разстоянието от
по-мощната
свръхнова



5.



от ресторанта ще се вижда върха на математическия хоризонт

$$\angle AOP = \angle TPB = \alpha$$

$$\angle APB + \angle AOP = \angle OPB + \angle AOB$$

$$\Rightarrow \angle AOB = \alpha$$

озн. $PB = a$

$$h \ll R \Rightarrow \tan \alpha = \frac{a}{R} \approx \alpha \text{ [rad]}$$

от Питоагорово теорема

$$a^2 + R^2 = (R+h)^2$$

$$a^2 + R^2 = R^2 + 2Rh + h^2$$

$$a = \sqrt{2Rh + h^2} = \sqrt{h(2R+h)} \approx$$

$$\approx 21 \sqrt{2R+h}$$

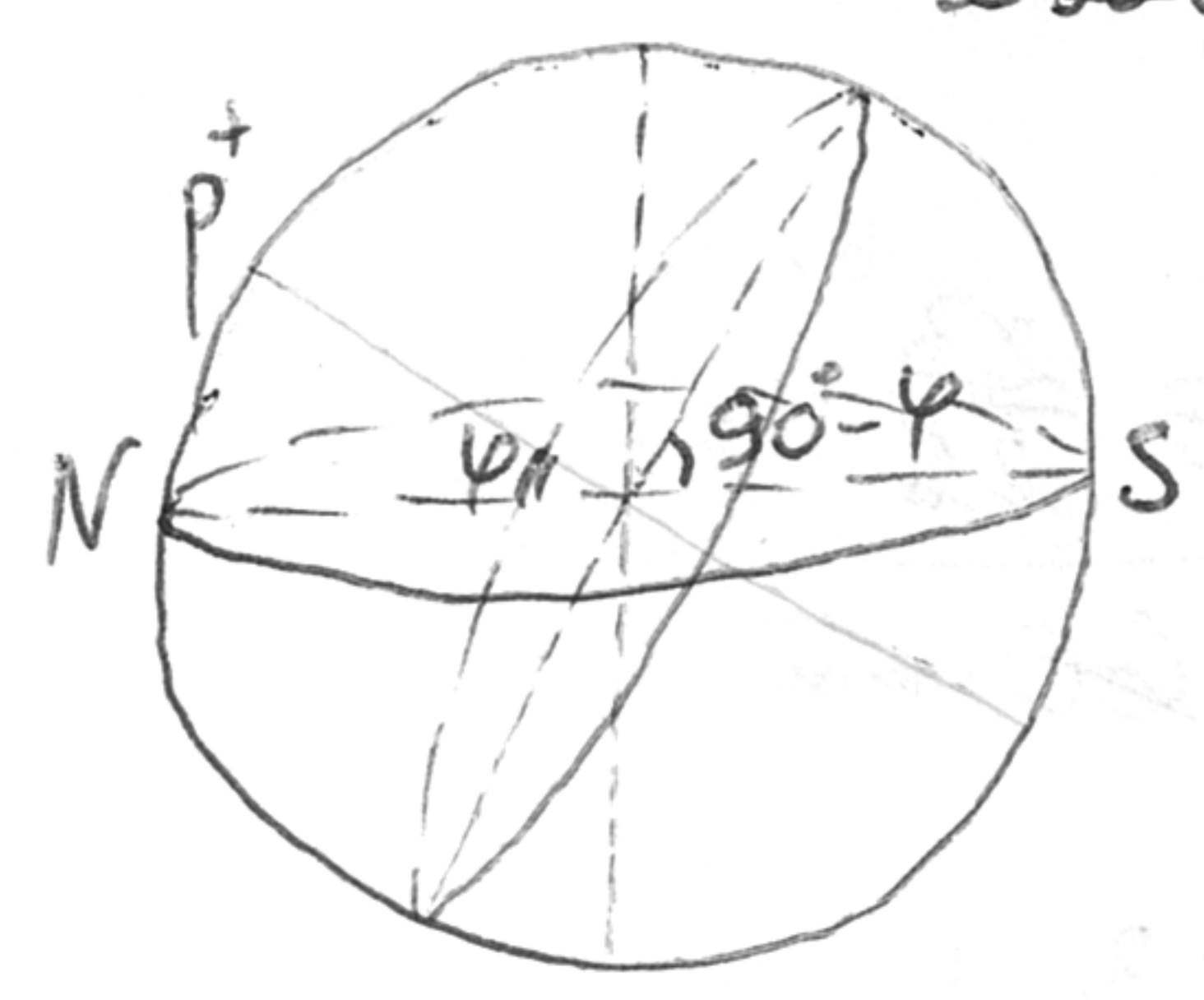
$$442 \approx 441 = 21^2$$

$$2R+h = 12742000 + 442 = 12742442$$

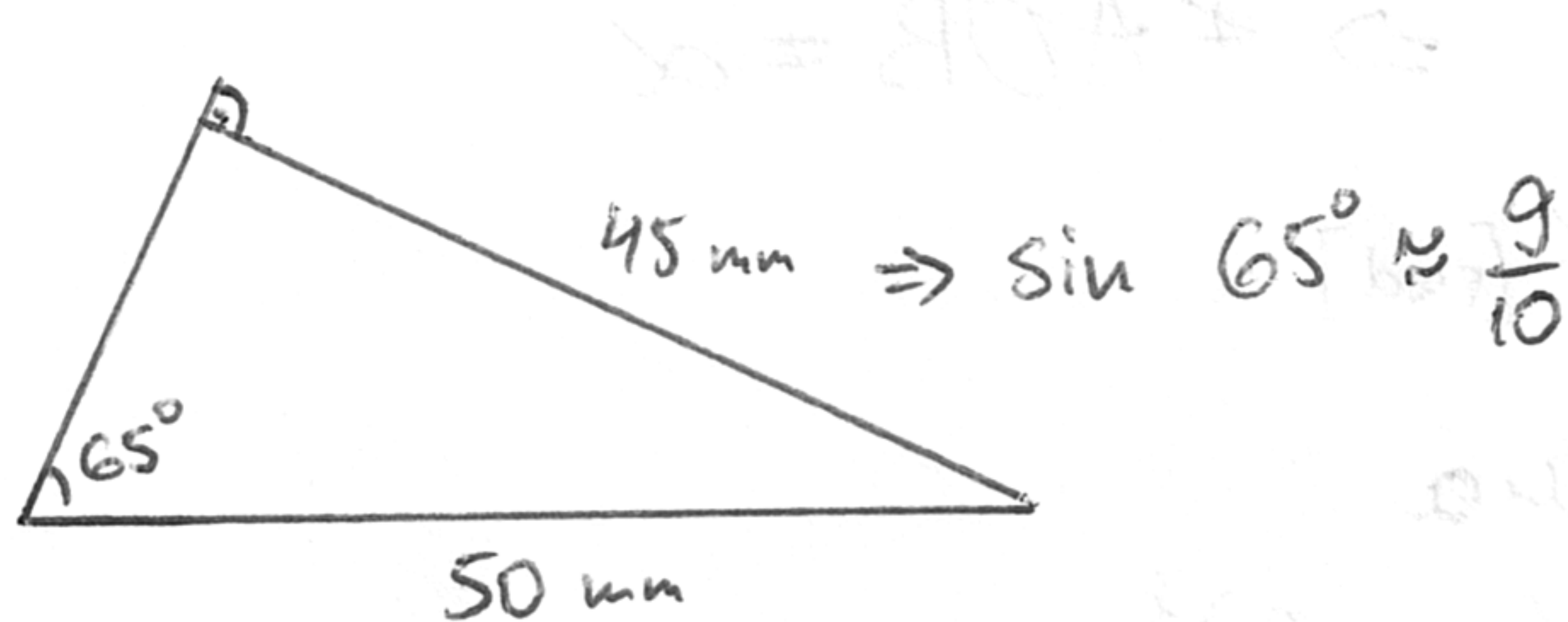
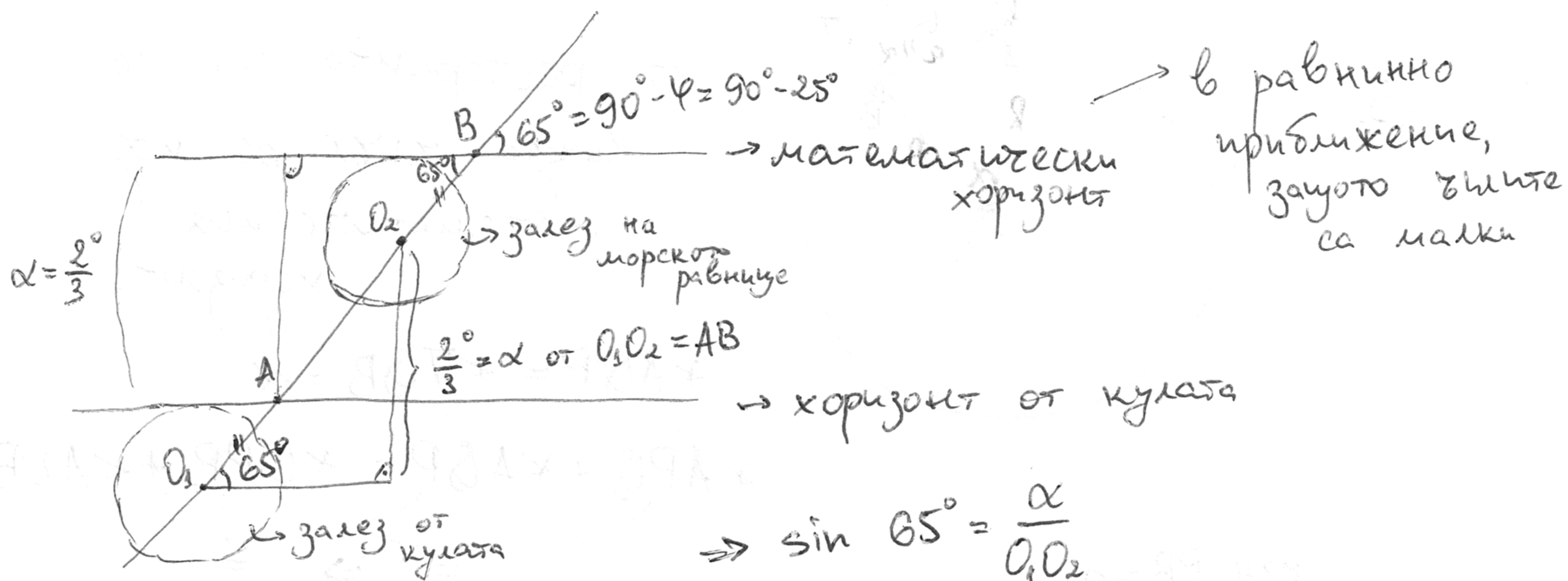
$$3570^2 = 12744900$$

$$\Rightarrow \sqrt{2R+h} \approx 3570$$

$$\alpha \approx \frac{75.48}{6370} \cdot 206265'' \approx \frac{206265}{8517}'' \approx 2430'' = \frac{2430}{60}' = 40.5' \approx \frac{2}{3}^\circ$$



Денонощното движение на Слънцето е успоредно на екватора - под ъгъл $90^\circ - \varphi = 65^\circ$ спрямо хоризонта.

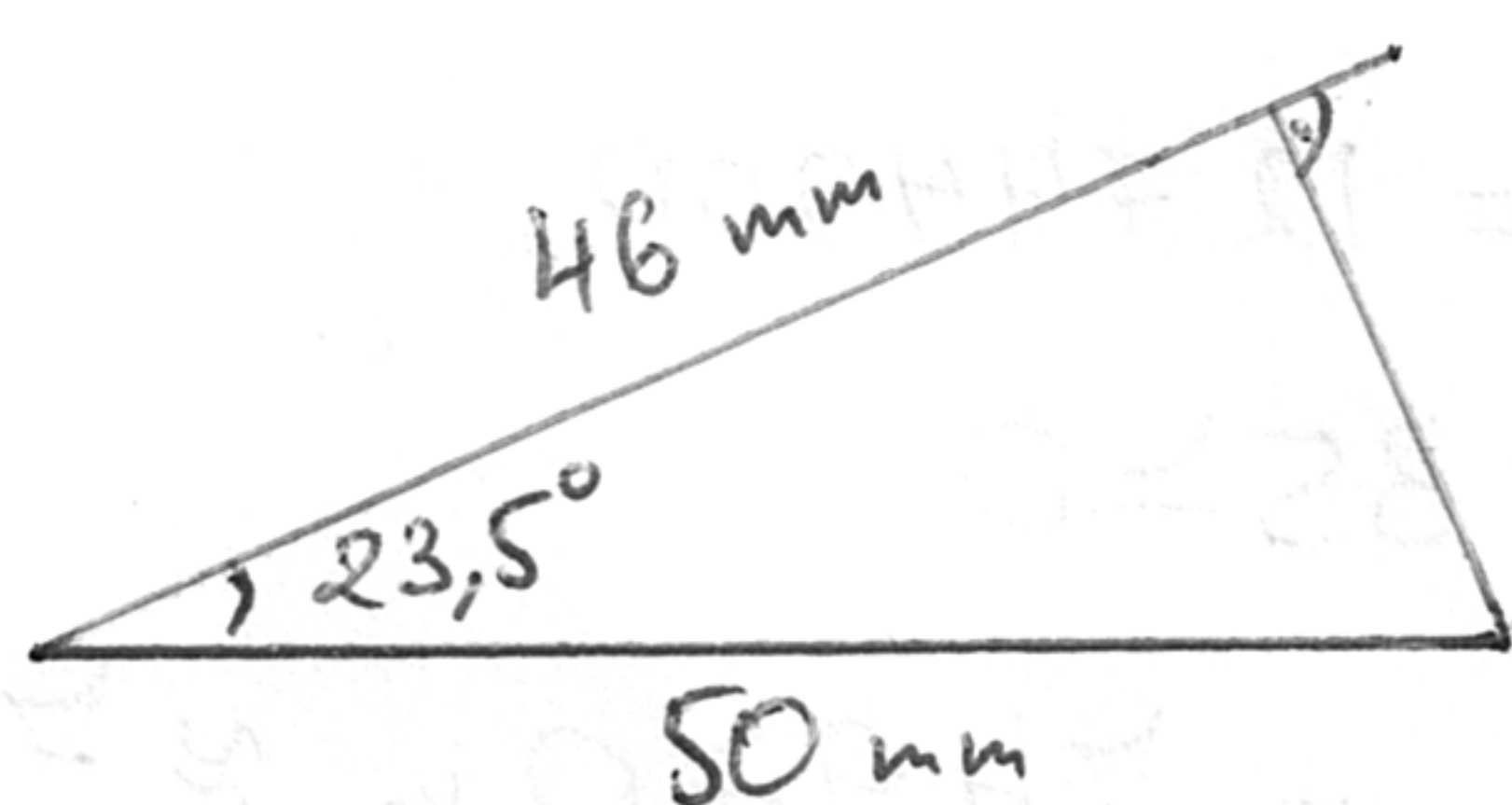


$$\frac{9}{10} = \frac{\frac{2}{3}}{O_1 O_2}$$

$$O_1 O_2 = \frac{2}{3} \cdot \frac{10}{9} = \frac{20}{27}$$

Толкова още трябва да изминем следващо, за да залезе

около следващото: $\delta_0 = 23,5^\circ$ - тогава за 24 h по ректасуекция не изминава 360° , а $360^\circ \cdot \cos \delta_0$, защото градусите стават "по-тънки" при по-голямо δ



$$\Rightarrow \cos 23,5^\circ \approx \frac{23}{25}$$

$$\omega_0 = \frac{O_1 O_2}{\Delta t} = \frac{360 \cdot \cos 23,5^\circ}{24 \text{ h}}$$

между залеж на морското равнище и залеж на кулата

$$\frac{\frac{20}{27}}{\Delta t \text{ min}} = \frac{\frac{360 \cdot \frac{23}{25}}{24 \cdot 60 \text{ min}}}{4}$$

$$\frac{20}{27 \Delta t} = \frac{23}{100}$$

$$\Delta t = \frac{2000}{621} = 3 \frac{137}{621} \approx 3,45 \text{ min}$$

Това е разлика при залеж и изгрев $\Rightarrow \Delta T = 2 \Delta t \approx \underline{\underline{6,9 \text{ min}}}$

1

$$6 \cdot 10^{24} = \frac{4}{3} \pi R^3 \rho$$

Чертова

$$6 \cdot 10^{24} = 4.600000000^3 \rho$$

$$10^6 = 4.6^2 \rho \quad 4.36 = 144$$

$$\rho = \frac{1000000}{144} = 6944.44 \text{ kg/m}^3$$

$$6 \cdot 10^{24} = 4.600000000^3 \rho \quad R_{\oplus} = 6000 \text{ km}$$

$$g = \frac{GM}{R^2} = \frac{GM_{\oplus}}{R_{\oplus}^2} \quad M = 100 M_{\oplus}$$

$$\frac{a^3}{T^2} = \frac{GM}{4\pi^2}$$

$$\frac{a_c^3}{T_c^2} = \frac{GM_{\oplus}}{4\pi^2}$$

$$\frac{a[a_c]}{T[T_c]} = M[M_{\oplus}]$$

$$\frac{a[a_c]}{1} = 100 \quad a[a_c] = \sqrt[3]{100}$$

45.45
+ 225
180
2025.45
10125
+ 8100
91125

47.47
+ 329
188
2209.47
15463
+ 8836
103823

46.46
+ 276
184
2116.46
12696
+ 8464
97336

465.465
+ 2325
2790
30225.465
151125
+ 181350
120900
14054625

60000
1790000

1700 = R
378000 173000

380 1700 = F
38 17

384400
6400
3844.6 = 23064

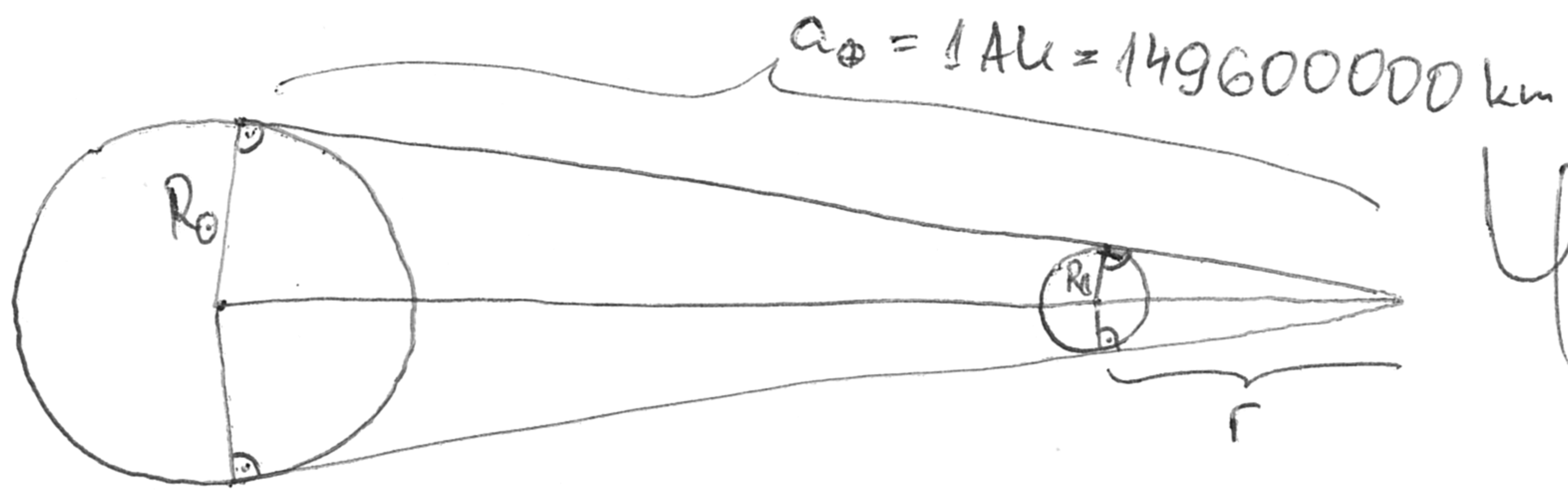
173.17 294100 =
1211 38
173 38
2941 38
3844.465

19220
23064 294100:38 =
15376 228
1788460 266

465.465
+ 2325 150
2790 114
30225.465 360
465.465 342
18

2325
+ 2790
1860
217225.465
1086125
+ 1203350
868900
100009625

3.

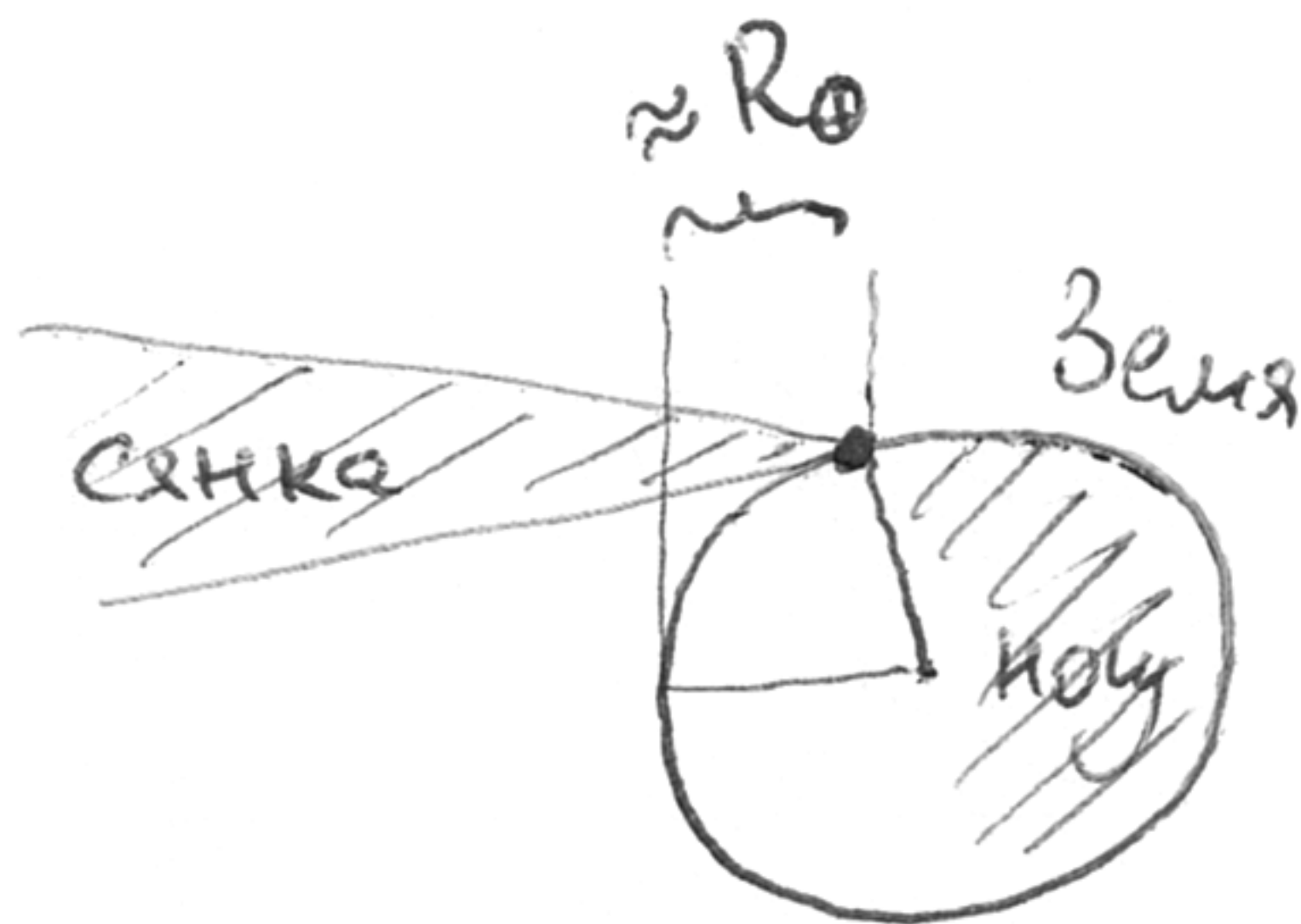


Черков

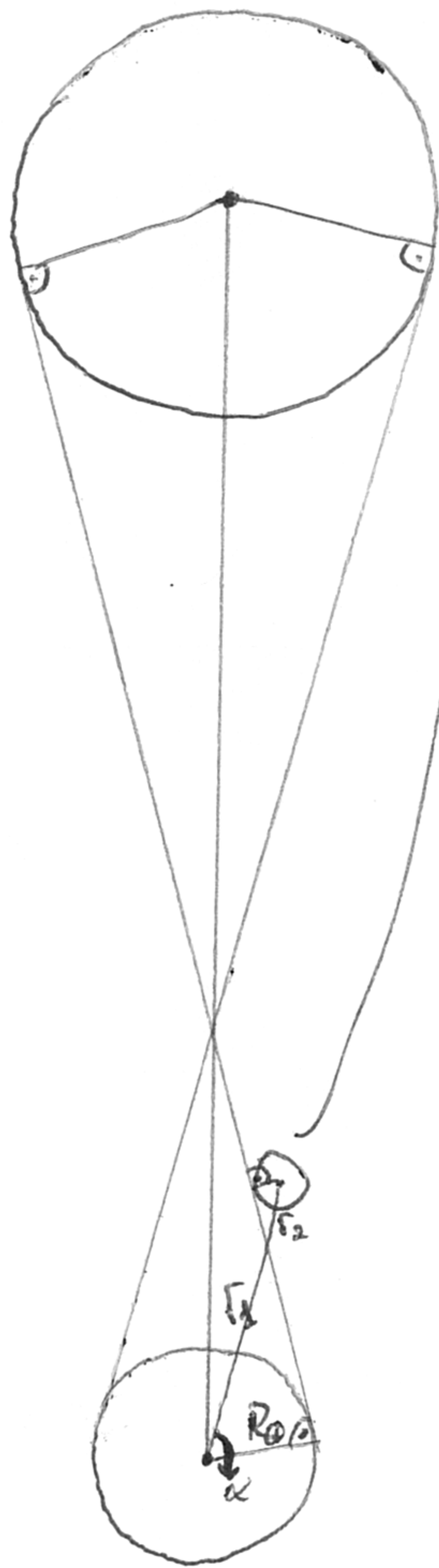
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след този момент в ъгъла започва тъмно затъмнение

$$\frac{r_1}{R_{\oplus}} = \frac{r_2}{R_{\oplus}}$$

$$\frac{R_{\oplus}}{R_{\oplus}} = \frac{6371}{1737} \approx 3,7$$

$$r_1 \approx 3,7 r_2$$

$$r_1 + r_2 = r \Rightarrow 4,7 r_2 = 372000$$

$$r_2 \approx 80000 \text{ km}$$

$$r_1 \approx 302000 \text{ km}$$

$$\Rightarrow \cos \alpha = \frac{R_{\oplus}}{r_1} \approx \frac{6400}{302000} \approx \frac{16}{730}$$

$$\frac{R_{\oplus}}{a_{\oplus}} = \rho \text{ [rad]}$$

$$\frac{R_{\oplus}}{a_{\oplus}} = \rho \text{ [rad]}$$

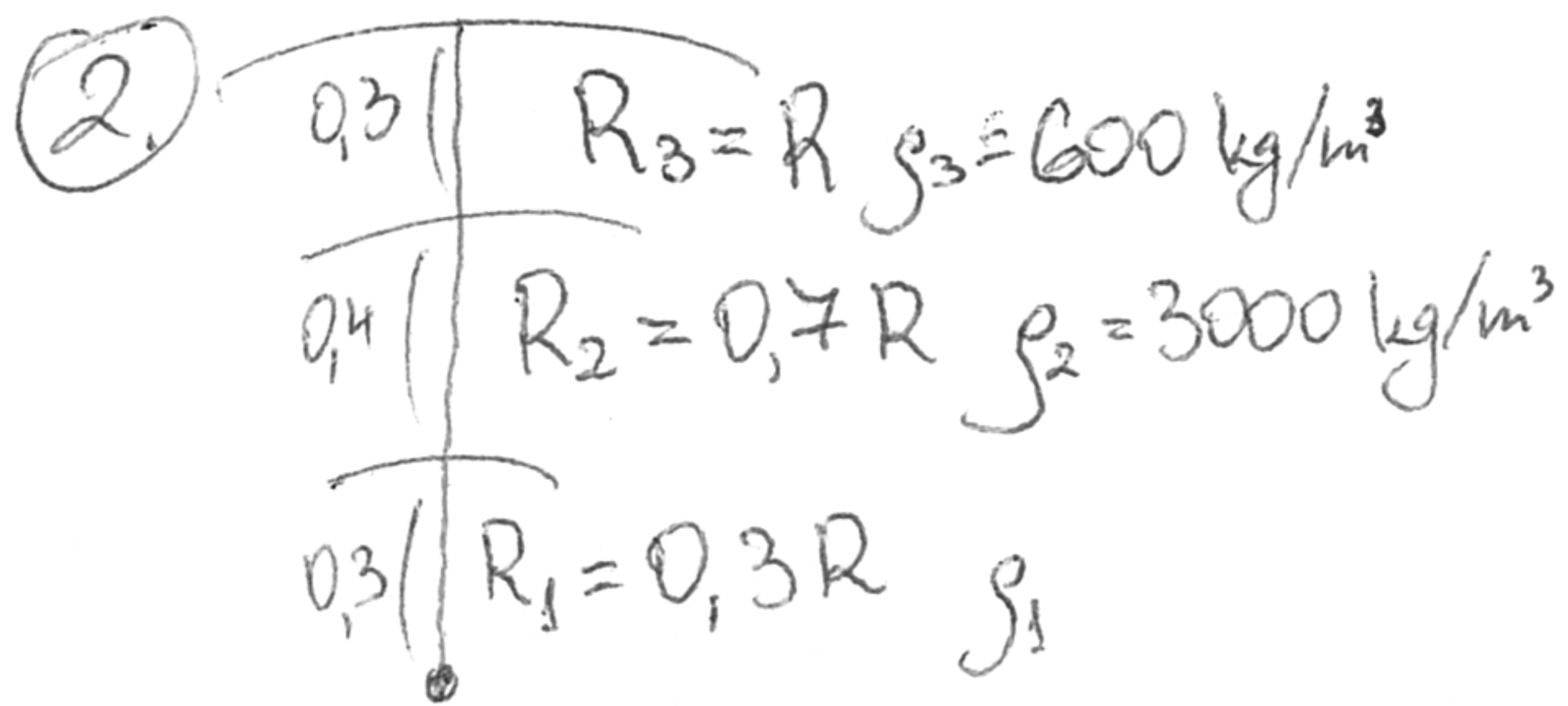
$$\frac{6400}{150000000} \cdot 200000 =$$

$$\frac{64}{1500} = \frac{128}{1500} = 4 \frac{4}{15}''$$

$$\frac{372000}{1737} \cdot \frac{200000}{6} = \frac{20000}{6} = 3333$$

2

Чернова



$$\rho = 1530 \text{ kg/m}^3$$

$$M = \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\pi (0,3R)^3 \rho_1 +$$

$$+ \frac{4}{3}\pi (0,7R)^3 \rho_2 - \frac{4}{3}\pi (0,3R)^3 \rho_2 +$$

$$+ \left(\frac{4}{3}\pi R^3 - \frac{4}{3}\pi (0,7R)^3 \right) \rho_3 =$$

$$= \frac{4}{3}\pi R^3 \left(0,3^3 \rho_1 + (0,7^3 - 0,3^3) \rho_2 + (1 - 0,7^3) \rho_3 \right)$$

$$\rho = 0,027 \rho_1 + (0,343 - 0,027) \rho_2 + (1 - 0,343) \rho_3$$

$$\frac{49,7}{343}$$

$$1530 = 0,027 \rho_1 + 0,316 \overset{3000}{\rho_2} + 0,657 \cdot 600$$

$$65,7 \cdot 6 =$$

$$1530 = 0,027 \rho_1 + 948 + 394,2$$

$$= 3942$$

$$1242,2$$

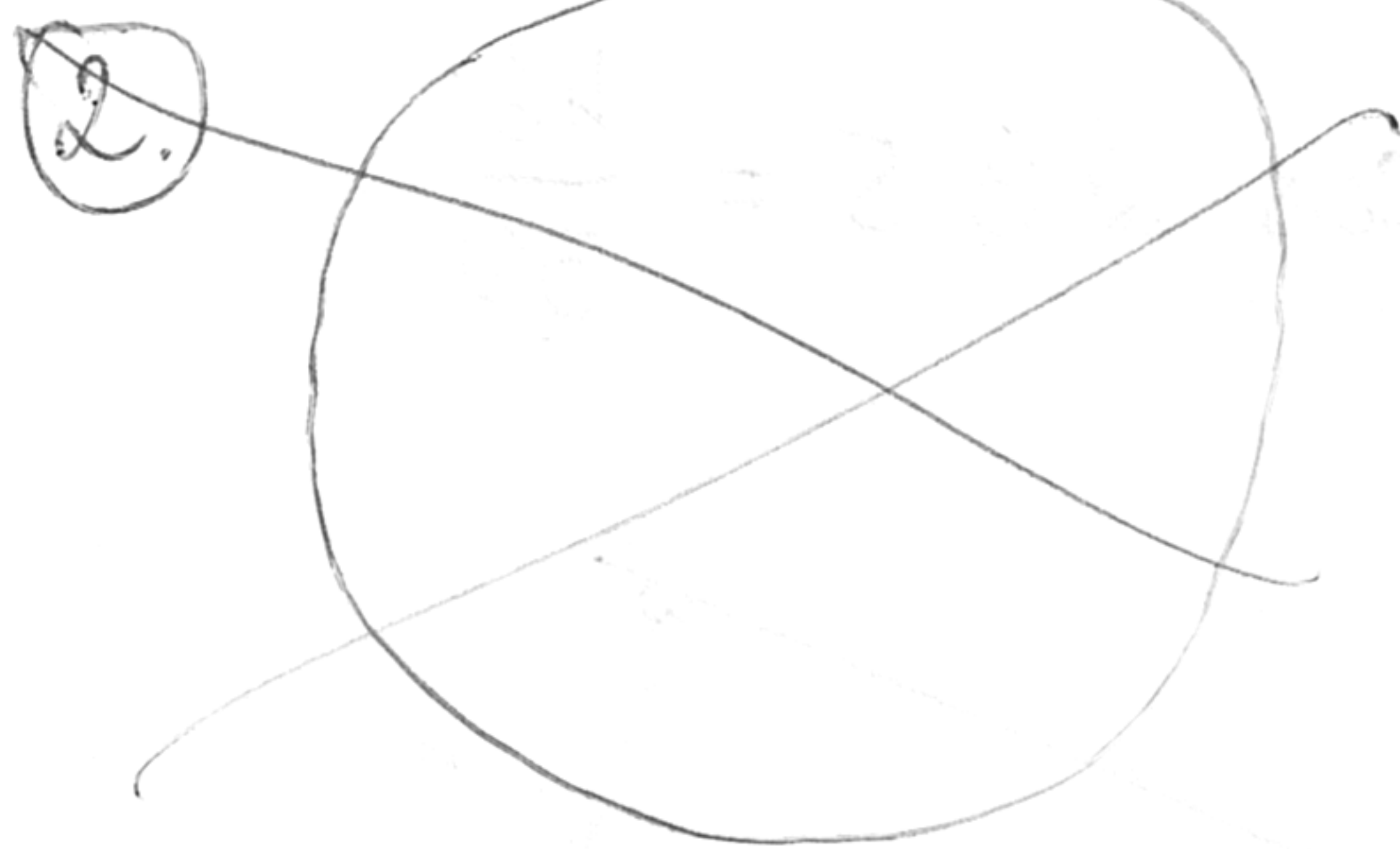
$$0,027 \rho_1 = 187,8$$

$$\rho_1 = \frac{187800}{27} = 6955,5$$

$$62600 : 9 = 6955,5$$

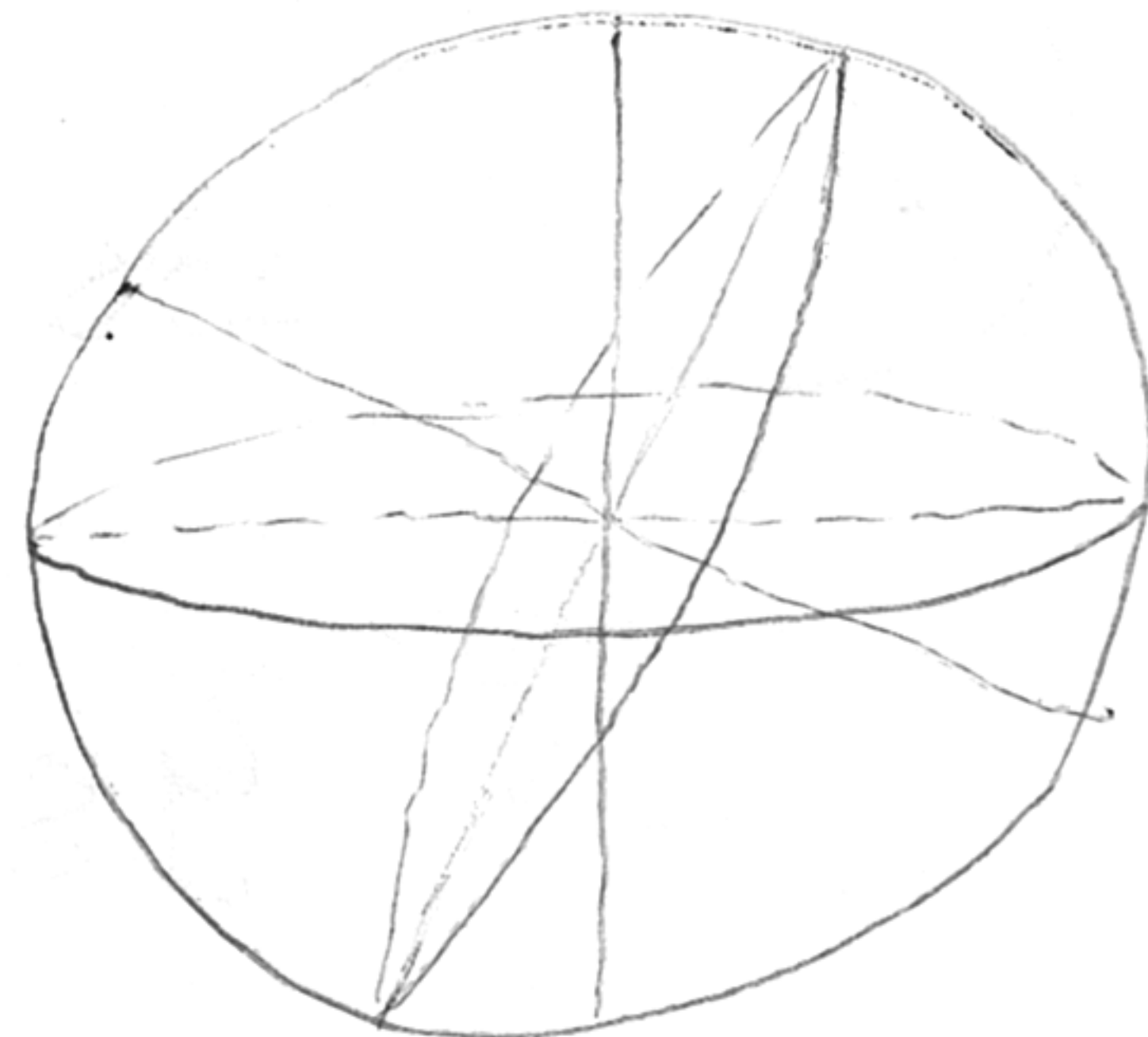
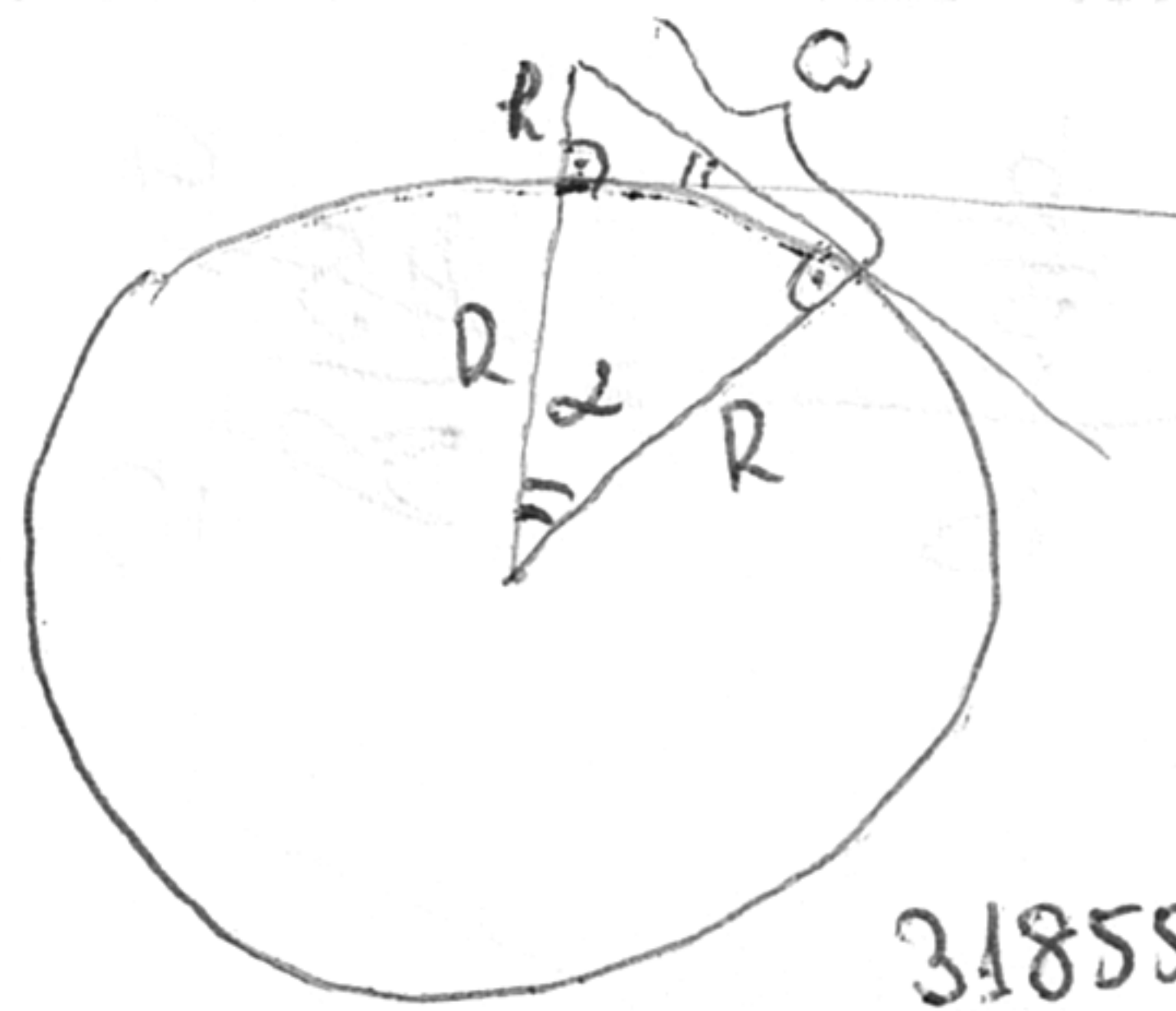
$$62095 : 9 = 6956,1$$

$$\begin{array}{r} 1530,0 \\ - 1342,2 \\ \hline 187,8 \end{array}$$



Чертова

5.



3185500
~~6371000~~
~~6371442~~
 11 3185721
 17 2427

11
~~220~~
~~3185500~~
 159275

~~206265 = 85~~

41253 : 17 = 2427

41253
 - 341

 72
 - 68

 45
 - 34

 113
 - 102

 11

1
 12742

12743
 12742

$\sin \alpha = \frac{a}{R} \approx \alpha [\text{rad}]$

$a^2 + R^2 = R^2 + 2Rh + h^2$

$a = \sqrt{(2R + h)h}$
 $h = 2 \sqrt{2Rh} =$

6371000.442
 12742
 25484
 25484

 2815982000

41253 : 17 = 2427
 41253
 - 341

 72
 - 68

 45
 - 34

 113
 - 102

 11

= 21.3570 = 12,742,442 4000
 = 74970 km

357.21

357
 + 414

 7497

a = 75 km

alpha = 206265

6370
 1275

 25585 17

3500 3500
 175
 + 105

 12250000

3600 3600
 216
 + 108

 12960000

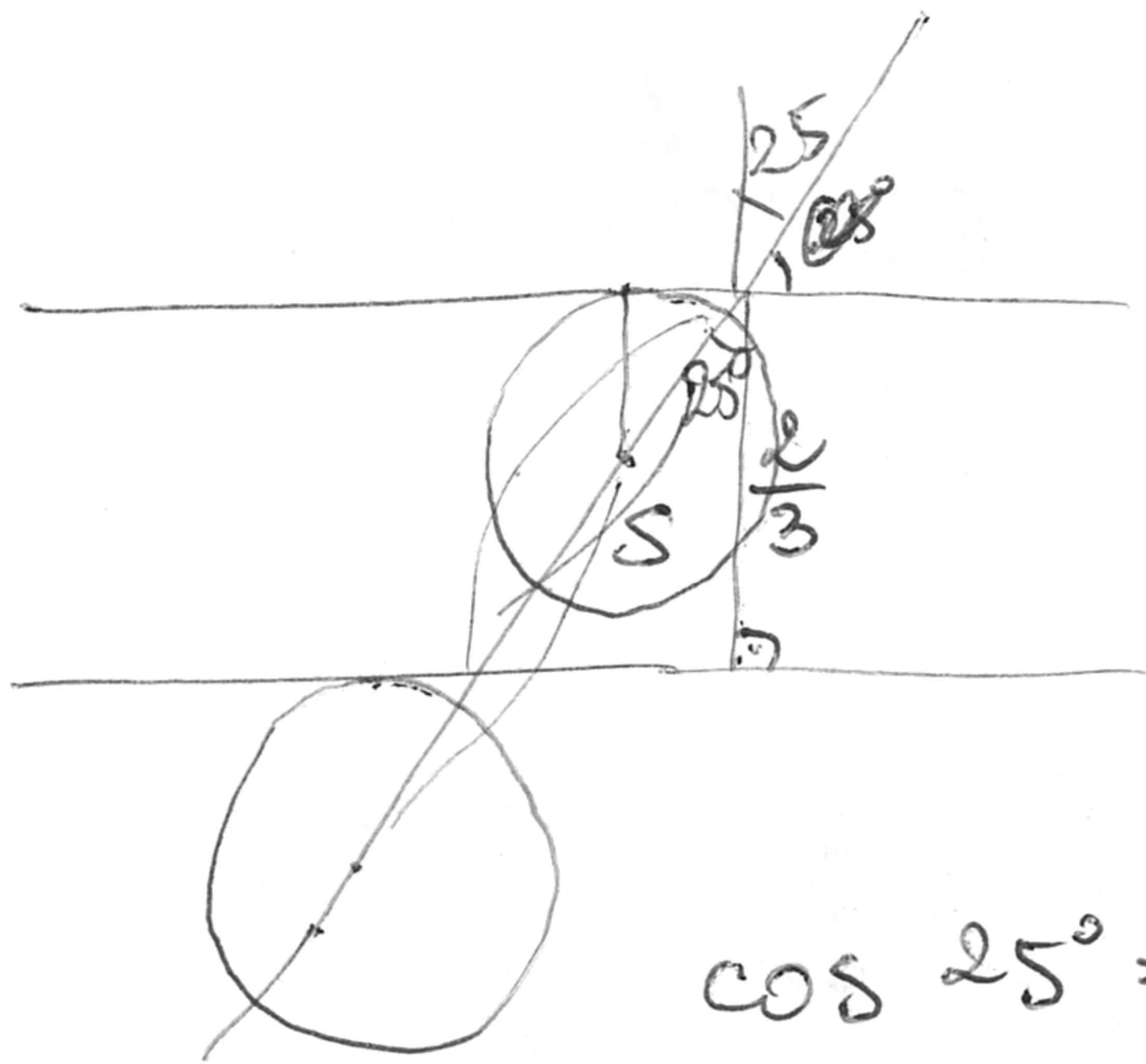
3570 3570
 2499
 + 1485
 + 1071

 12,744,900

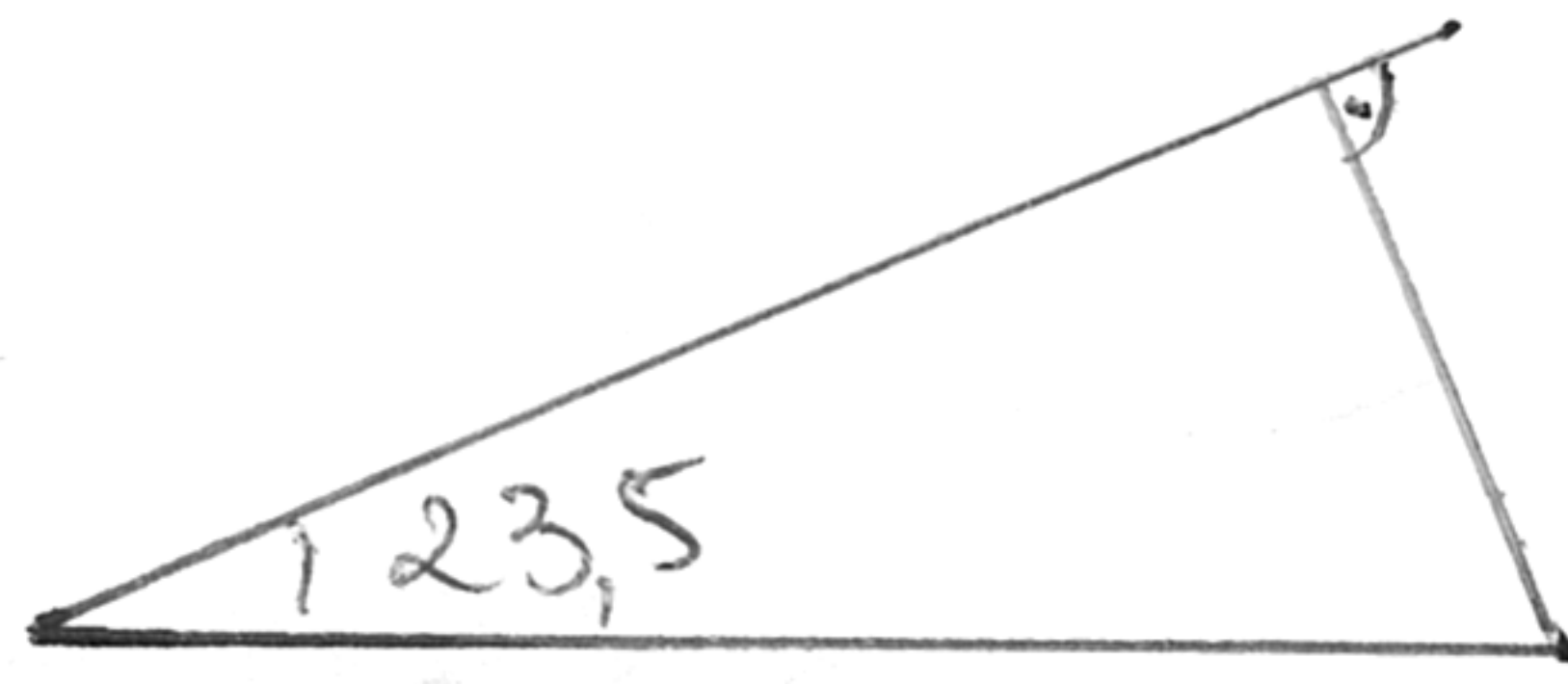
3570.21
 357
 + 414

 74970

2427 : 60 = 40.5' approx 2/3



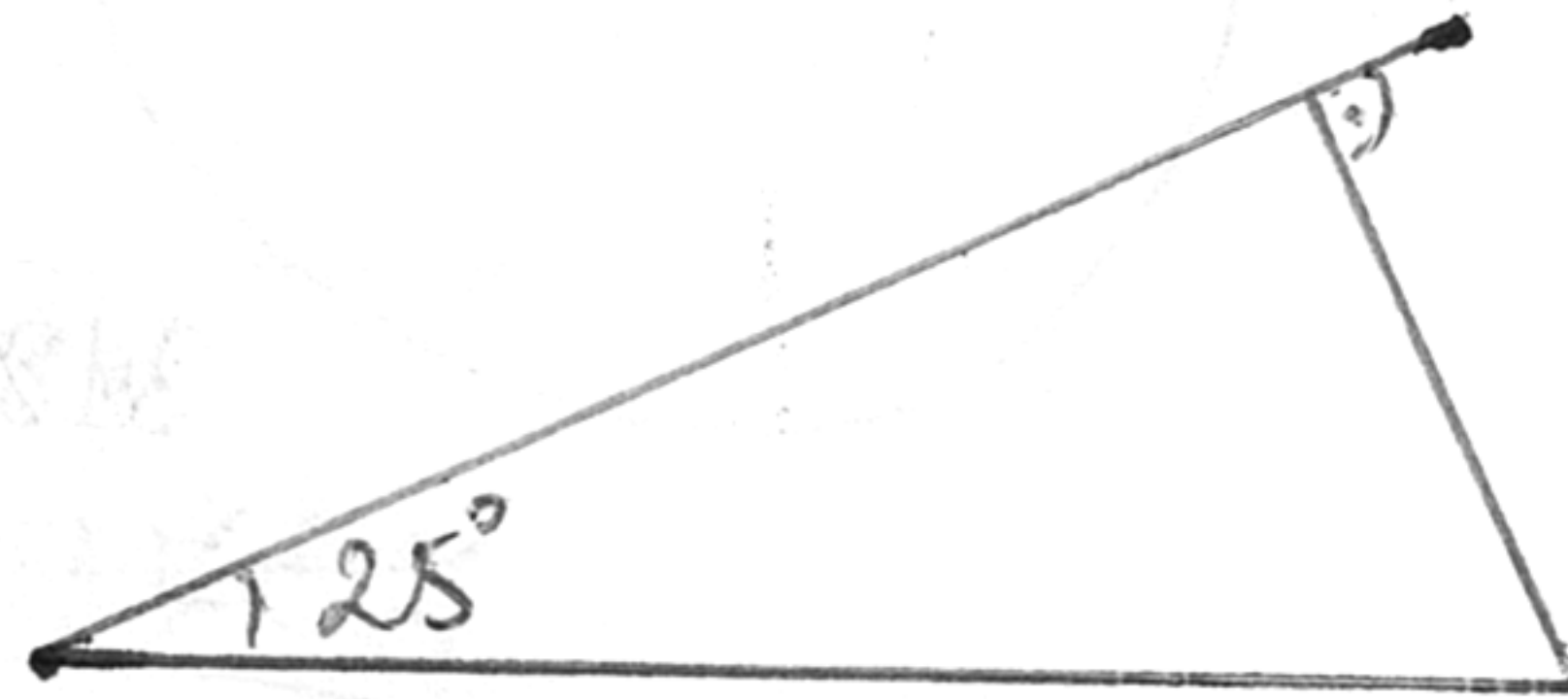
$$\cos 23,5 = \frac{46}{50}$$



$$\cos 25^\circ = \frac{\frac{2}{3}}{S} = \frac{45^9}{50 \cdot 10}$$

$$9S = \frac{20}{3}$$

$$S = \frac{20}{27}$$



360 ~~cos~~ $\frac{9}{10} = 360^\circ$
 $3.621 = 1863$
 $\frac{S}{24.60} = \frac{20}{324}$
 $\frac{2000}{1863} = 1.37$
 $\frac{20}{27} \cdot 24 = 324.8$

$$\frac{27.23}{81} + \frac{54}{624}$$

$$137.4 = \frac{20}{27} \cdot 24 = 324.8$$

$$137.5 = \frac{800}{61} = S = \frac{137.45}{685} + \frac{548}{6165}$$

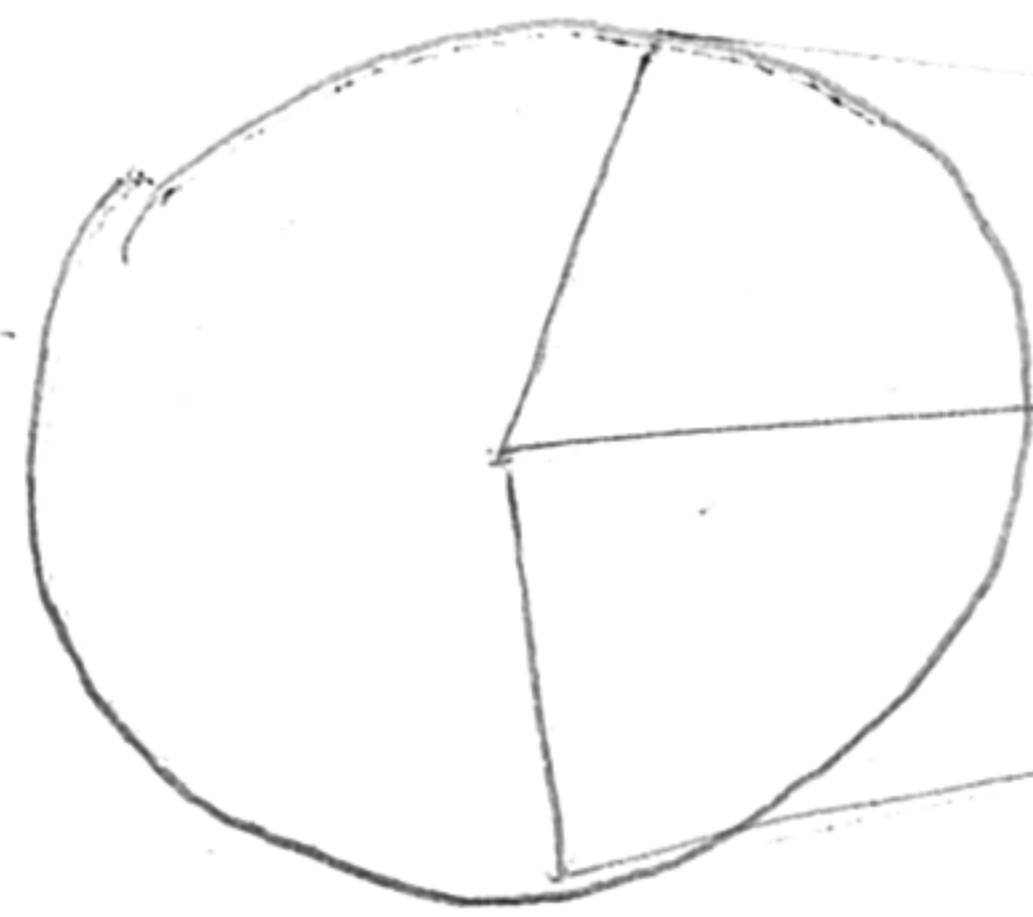
$$3 \frac{17}{61} = 3,3 \text{ min} \cdot 2$$

$$200 - 183 = 17$$

$$17 : 61 = 0,27027 = 0,6$$

$$\begin{array}{r} 170 \\ 122 \\ \hline 480 \\ 427 \\ \hline 53 \end{array}$$

3.



$R_0 = 696000 \text{ km}$

$R_c = 1737 \text{ km}$

Уртраба
1496.1737
10472
+ 4488
10472
1496

2598552

$\frac{696000}{1737} = \frac{1737}{r}$
~~18000000~~
496

$696r = 259855200$

$259855200 : 696 = 373350$

$\frac{259855200}{696}$
2088

5105
4872

23035
2088

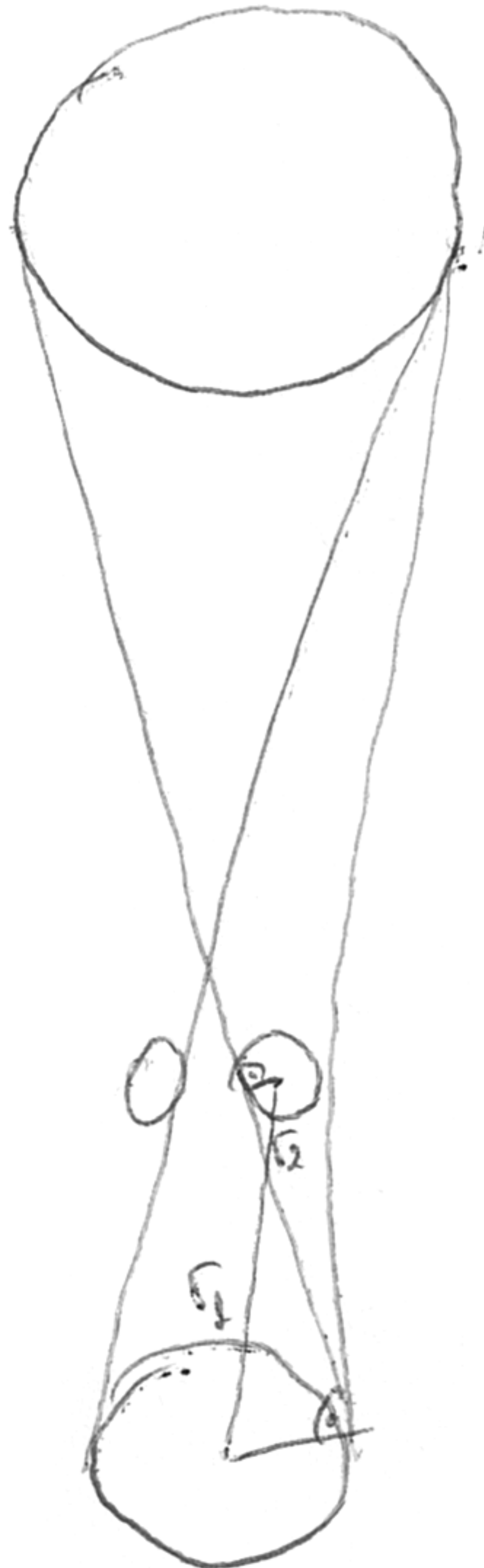
2452
2088

43840

$\frac{400000}{384400} = 1.040845473$
3948
156

3844
4922

961



$\frac{6371}{1737} = \frac{r_1}{r_2}$

$\frac{6400}{1700} = 3 \frac{13}{17} = 3,7$

$\frac{384400}{3844} = 100$
3844

26908
3844

3844

44948

$\frac{150000}{1737} = 86.356362124$
248

150000
1737

86356362124



$4,7 \cdot r_2 = 372000$

$372 : 47 = 7,914893617$

$47 r_2 = 372000$
80000

$\frac{12159}{5211} = 2,333333333$
12159

5211

64269

$\frac{5211}{10422} = 0,5$
5211

10422

$7,3 \cdot 2 = 14,6$

$\frac{910}{16} = 56,875$
80

110

96

14

$\frac{730}{16} = 45,625$
64

90

80

10

$$\textcircled{4} \sqrt[5]{32} = 2$$

$$R_1 \approx \sqrt[5]{E} \cdot t^{2/5}$$

$$R_2 \approx \sqrt[5]{32 \cdot E} \cdot t^{2/5}$$

$$\frac{R_1}{R_2} = \frac{\sqrt[5]{E} \cdot t^{2/5}}{2 \cdot \sqrt[5]{E} \cdot t^{2/5}} = \frac{1}{2}$$

Упрощава

$$R_2 = 2R_1$$

$$\Rightarrow 200 \text{ pc}$$