XXIX Санкт-петербургска олимпиада по астрономия

Теоретичен тур

6 февруари 2022г.

Задача 1.

На полюса: $T_p = 2\pi\sqrt{\frac{l}{g_0}}$, $g_0 = \frac{\gamma M}{R^2}$

$P = 10h$ $\omega = \frac{2\pi}{P}$

На екватора: $g_{eff} = g_0 - \omega^2 R$

$$T_e = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{g_0 - \omega^2 R}}$$

$$\eta = \frac{T_e - T_p}{T_p} = \frac{T_e}{T_p} - 1 =$$

$$= \frac{\cancel{2\pi}\sqrt{\frac{\cancel{l}}{g_0-\omega^2 R}}}{\cancel{2\pi}\sqrt{\frac{\cancel{l}}{g_0}}} - 1 = \left(\frac{g_0}{g_0-\omega^2 R}\right)^{\frac{1}{2}} - 1$$

$$\eta = \left(\frac{g_0 - \omega^2 R}{g_0}\right)^{-\frac{1}{2}} - 1 = \left(1 - \frac{\omega^2 R}{g_0}\right)^{-\frac{1}{2}} - 1 \approx$$

$$\approx \cancel{1} + \frac{1}{2}\frac{\omega^2 R}{g_0} - \cancel{1} \qquad (\eta = 2\% << 1)$$

$$\eta = \frac{\omega^2 R}{2 g_0}$$

На височина $h = 30\,km$:

$$g_h = \frac{\gamma M}{(R+h)^2} = g_0 \cdot \frac{R^2}{(R+h)^2}$$

$$T_h = T_e = 2\pi\sqrt{\frac{l}{g_h}} = 2\pi\sqrt{\frac{l}{g_0 \cdot \frac{R^2}{(R+h)^2}}} =$$

$$= 2\pi\sqrt{\frac{l}{g_0}} \cdot \frac{R+h}{R} =$$

$$= T_p\left(1 + \frac{h}{R}\right)$$

$$T_h = T_e \Rightarrow \eta = \frac{T_h - T_p}{T_p} = \frac{\cancel{T_p}\left(1 + \frac{h}{R}\right) - T_p}{\cancel{T_p}} =$$

$$= \cancel{1} + \frac{h}{R} - \cancel{1}$$

$$\Rightarrow \eta = \frac{h}{R}$$

$$R = \frac{h}{\eta} = \frac{130\,km}{0,02} = \frac{75}{0,01} = 7500\,km$$

$$g_0 = \frac{\omega^2 R}{2\eta} = \frac{\cancel{4}^2\pi^2}{P^2} \cdot \frac{R}{\cancel{2}_1\eta} = \frac{2\pi^2 R}{\eta P^2} =$$

$$= \frac{2 \cdot 3,14^2 \cdot 7,5 \cdot 10^6\,m}{0,02 \cdot (100 \cdot 3600\,s)^2} \approx$$

$$\approx \frac{\cancel{2} \cdot 10 \cdot 7,5 \cdot \cancel{10^6}}{\cancel{2} \cdot \cancel{10^{2}} \cdot \cancel{10^{2}} \cdot 3,6^2 \cdot \cancel{10^{6}}} \approx \frac{75}{13}$$

$$g_0 \approx 5,8\,m/s^2$$

Максималната скорост на движение без двигатели е втора космическа

$$V = \sqrt{\frac{2\gamma M}{R}} = \sqrt{\frac{2\gamma M}{R^2} \cdot R} = \sqrt{2 g_0 R} =$$

$$= \sqrt{2 \cdot 5,8 \cdot 7500} = \sqrt{5,8 \cdot 15000} =$$

$$= \sqrt{\underbrace{5,8 \cdot 1,5}_{\approx 9}} \cdot 10^2 \approx 3 \cdot 10^2\,m/s$$

Задача 2.

Щом могат да се розличат поотделно, те не са в състава на тези двойни системи, което отхвърля наличието и на гигант, и на джудже в една система. Тогава те са: гигант с гигант и джудже с джудже. Наличието на два сини гиганта в една ~~[illegible]~~ система е много малко вероятно => системите са:

1. Червен гигант и червен гигант.
2. Синьо джудже и синьо джудже.

Червените гиганти са етап, който еволюционно предхожда джуджетата.

=> Система 2 е по-стара

Задача 3.

Маса на Млечния път: $M_{MW} \sim 10^{42}$ кg

Маса на Местната група: $M \sim 10 M_{MW} \sim 10^{43}$ кg

На порядък лъчевата скорост на орбитално движение на галактиката е:

$$V_r(r) \sim \sqrt{\frac{\gamma M}{r}}$$

~~[illegible]~~ Ще спре да се наблюдава синьо отместване когато:

$$V_r(r) - Hr = 0$$

$$\sqrt{\frac{\gamma M}{r}} - Hr = 0$$

$$\frac{\gamma M}{r} = H^2 r^2$$

$$r = \sqrt[3]{\frac{\gamma M}{H}}$$

$$H \approx 69 \frac{km/s}{Mpc}$$

$$1 Mpc = 10^6 . 206265 AU \approx 10^6 . 2 . 10^5 . 150 . 10^6 km = $$
$$= 2 . 10^{11} . 1{,}5 . 10^{11} m = 3 . 10^{22} m$$

$$H = \overset{23}{\cancel{6}9} \cdot \frac{10^{3}\ m/s}{\cancel{3} \cdot 10^{22}\ m} = 23 \cdot 10^{-19}\ s^{-1}$$

$$r = \sqrt[3]{\frac{6{,}67 \cdot 10^{-11} \cdot 10^{43}}{2{,}3 \cdot 10^{-18}}} \approx \sqrt[3]{3 \cdot 10^{72}}$$

$$r \sim 10^{24}\ m \sim 100\ Mpc$$

Задача 4.

$$\lambda_0 = 6263\ \text{Å}$$

$$\frac{\Delta\lambda}{\lambda_0} = \frac{V_1}{c}$$

$$V_1 = c\,\frac{\Delta\lambda}{\lambda_0} = 3.10^5 \cdot \frac{0,46}{6263} = 3.\cancel{10^5} \cdot 7,7.\cancel{10^5}$$

$V_1 \approx 23\ km/s$ – орбитална скорост на бялото джудже

Бялото джудже със сигурност се вижда във филтър V. Възможно е другата звезда да не се и от това зависи дали минимумите ще се наблюдават през половин период ($T = 2t = 1y$) или един период ($T = t = 0,5y$).

$$V_1 = \frac{2\pi a_1}{T} \Rightarrow a_1 = \frac{V_1 T}{2\pi}$$

$$\frac{a^3}{T^2} = \frac{\gamma(M_1 + M_2)}{4\pi^2} \Rightarrow a = \sqrt[3]{\frac{\gamma(M_1 + M_2)T^2}{4\pi^2}}$$

M_1 – бяло джудже; M_2 – другата компонента

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$$\begin{cases} M_1 a_1 = M_2 a_2 \\ a_1 + a_2 = a \end{cases} \Rightarrow a_1 = \frac{M_2}{M_1+M_2} a$$

$$\frac{v_1 T}{2\pi} = \frac{M_2}{M_1+M_2} \sqrt[3]{\frac{\varkappa(M_1+M_2)T^2}{4\pi^2}}$$

$$\cancel{\frac{v_1 T}{2\pi} = M_2 \left(\frac{T^2}{M_1+M_2}\right)^{2/2}} \quad \cancel{\frac{v_1 T}{2\pi} = M_2 \cdot \left(\frac{T}{M_1+M_2}\right)^{2/3}}$$

~~Горна граница:~~ $\cancel{M_2 >> M_1}$

$$\cancel{\frac{v_1 T}{2\pi} \approx M_2 \cdot \frac{T}{M_2}^{2/3} = M_2^{1/3} \cdot T^{2/3}}$$

$$\cancel{M_2^{1/3} = \frac{v_1 T^{1/3}}{2\pi}}$$

$$\cancel{M_2 = \frac{v_1^3 T}{8\pi^3}}$$

$\cancel{T = 1y: \quad M_2 = 23^3 \cdot 1.}$

$$\frac{v_1 T}{2\pi} = \frac{M_2}{M_1+M_2} \cdot \sqrt[3]{\frac{\varkappa(M_1+M_2)T^2}{4\pi^2}}$$

$$\frac{v_1^3 T^{\cancel{3}}}{{}_2 8\pi^{\cancel{3}}} = \frac{M_2^3}{(M_1+M_2)^{\cancel{3}\,2}} \cdot \frac{\varkappa\,\cancel{(M_1+M_2)}\cancel{T^2}}{4\cancel{\pi^2}}$$

$$\frac{v_1^3 T}{2\pi} = \frac{\gamma M_2^3}{(M_1+M_2)^2}$$

Горна граница: $M_2 \gg M_1$

$$\frac{v_1^3 T}{2\pi} \approx \frac{\gamma M_2^{\cancel{3}}}{\cancel{M_2^2}}$$

$$M_2 = \frac{v_1^3 T}{2\pi\gamma} \Rightarrow M_2[M_\odot] = \left(\frac{v_1}{v_\oplus}\right)^3 \cdot \frac{T}{T_\oplus}$$

$v_\oplus \to 30\,km/s$, $T_\oplus \to 1y$

$T = 0,5\,y$: $M_2 = \left(\frac{23}{30}\right)^3 \cdot \frac{0,5}{1} = (0,77)^3 \cdot 0,5$

$$M_2 \approx 0,5 \cdot 0,5 \approx 0,2\,8\,M_\odot$$

Това съответсва на звезда от спектрален клас К–М, т.е. тип кафяво джудже. Те не излъчват (силно) във видимия диапазон. По-разумна горна граница съответства на $T = 0,5\,y$.

$T = 1,0\,y$: $M_2 \approx 0,55\,M_\odot$

Очевидно условието $M_2 \gg M_1$ не е спазено, така че това е само оценка. $M_{2max} \approx 0,5\,M_\odot$

Долна граница: $M_2 << M_1$

$$\frac{v_1^3 T}{2\pi} \approx \frac{\gamma M_2^3}{M_1^2}$$

$$M_2 = \sqrt[3]{\frac{v_1^3 T}{2\pi} \cdot M_1^2}$$

$$M_2[M_\odot] = \sqrt[3]{\left(\frac{v_1}{v_\oplus}\right)^3 \frac{T}{T_\oplus} \cdot M_1[M_\odot]^2} =$$

$T = 0{,}5y$: $M_2 \approx \sqrt{0{,}55 \cdot 0{,}5 \cdot 1} = 0{,}45 M_\odot$

$(M_1 \sim 1 M_\odot)$

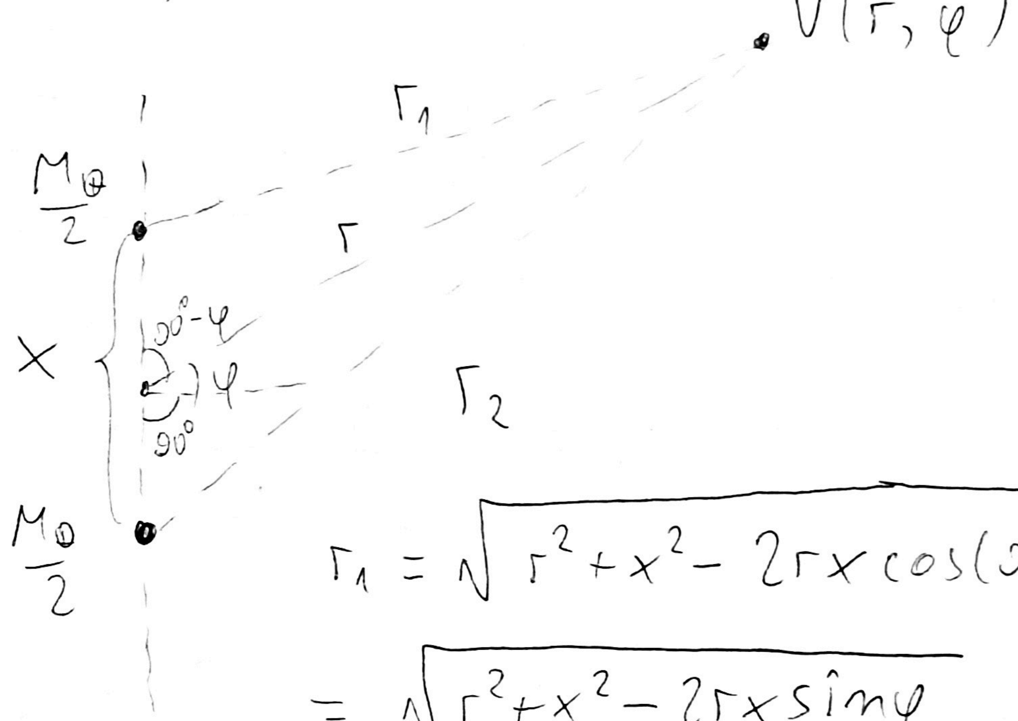
$T = 1y$: $M_2 = 0{,}63 M_\odot$

Очевидно анализът ни не е точен, но се вижда, че звездата трябва да бъде малка ($M \sim 0{,}5 M_\odot$), т.е. да е кафяво джудже

Ориентировъчни граници: $0{,}2 \div 0{,}6 M_\odot$

Задача 5.

Двете маси трябва да бъдат разположени симетрично спрямо центъра на Земята.



$$r_1 = \sqrt{r^2+x^2-2rx\cos(90^\circ-\varphi)} = \sqrt{r^2+x^2-2rx\sin\varphi}$$

$$r_2 = \sqrt{r^2+x^2-2rx\cos(90^\circ+\varphi)} = \sqrt{r^2+x^2+2rx\sin\varphi}$$

$$V(r,\varphi) = \frac{\gamma\frac{M_\oplus}{2}}{r_1} + \frac{\gamma\frac{M_\oplus}{2}}{r_2} =$$

$$= \frac{\gamma M_\oplus}{2}\left\{\frac{1}{\sqrt{r^2+x^2-2rx\sin\varphi}} + \frac{1}{\sqrt{r^2+x^2+2rx\sin\varphi}}\right\} =$$

$$= \frac{\gamma M_\oplus}{2} \left\{ \frac{1}{\sqrt{r^2\cos^2\varphi + (x^2 + 2rx\sin\varphi + r^2\sin^2\varphi)}} + \right.$$

$$\left. + \frac{1}{\sqrt{r^2\cos^2\varphi + (x^2 - 2rx\sin\varphi + r^2\sin^2\varphi)}} \right\} =$$

$$= \frac{\gamma M_\oplus}{2} \left\{ \frac{1}{\sqrt{r^2\cos^2\varphi + (x + r\sin\varphi)^2}} + \frac{1}{\sqrt{r^2\cos^2\varphi + (x - r\sin\varphi)^2}} \right\} =$$

$$= \frac{\gamma M_\oplus}{2r\cos\varphi} \left\{ \frac{1}{\sqrt{1 + \left(\frac{x + r\sin\varphi}{r\cos\varphi}\right)^2}} + \frac{1}{\sqrt{1 + \left(\frac{x - r\sin\varphi}{r\cos\varphi}\right)^2}} \right\}$$

$$(1+x)^n \approx 1 + nx + \frac{n(n-1)}{2} x^2$$

$$\Rightarrow V(r,\varphi) \approx \frac{\gamma M_\oplus}{2r\cos\varphi} \left\{ 1 - \frac{1}{2}\left(\frac{x + r\sin\varphi}{r\cos\varphi}\right)^2 + \right.$$

$$+ \frac{-\frac{1}{2}\left(-\frac{1}{2} - 1\right)}{2} \left(\frac{x + r\sin\varphi}{r\cos\varphi}\right)^4 + 1 - \frac{1}{2}\left(\frac{x - r\sin\varphi}{r\cos\varphi}\right)^2 +$$

$$\left. + \frac{3}{8}\left(\frac{x - r\sin\varphi}{r\cos\varphi}\right)^4 \right\}$$

$$V(r,\varphi) = \frac{\gamma M_{\oplus}}{2 r \cos\varphi}\left\{ 2 - \frac{1}{2} \cdot \frac{(x + r\sin\varphi)^2 + (x - r\sin\varphi)^2}{(r\cos\varphi)^2} + \right.$$

$$\left. + \frac{3}{8} \cdot \frac{(x + r\sin\varphi)^4 + (x - r\sin\varphi)^4}{(r\cos\varphi)^4} \right\} =$$

$$\cancel{= \frac{\gamma M_{\oplus}}{2 r \cos\varphi}\left\{ 2 - \frac{1}{2} \cdot x^2 + r^2 \sin^2\varphi \right.}$$

$$V(r,\varphi) = \frac{\gamma M_{\oplus}}{2r}\left\{ \frac{1}{\sqrt{1 + \frac{x^2}{r^2} - \frac{2x}{r}\sin\varphi}} + \frac{1}{\sqrt{1 + \frac{x^2}{r^2} + \frac{2x}{r}\sin\varphi}} \right\} \approx$$

$$\approx \frac{\gamma M_{\oplus}}{2r}\left\{ 1 - \frac{1}{2}\left(\frac{x^2}{r^2} - \cancel{\frac{2x}{r}\sin\varphi} \right) + \frac{3}{8}\left(\frac{x^2}{r^2} - \frac{2x}{r}\sin\varphi \right)^2 + \right.$$

$$\left. + 1 - \frac{1}{2}\left(\frac{x^2}{r^2} + \cancel{\frac{2x}{r}\sin\varphi} \right) + \frac{3}{8}\left(\frac{x^2}{r^2} + \frac{2x}{r}\sin\varphi \right)^2 \right\} =$$

$$= \frac{\gamma M_{\oplus}}{2r}\left\{ 2 - \frac{x^2}{r^2} + \frac{3}{8}\left(\frac{x^4}{r^4} - \cancel{\frac{4x^3}{r^3}\sin\varphi} + \frac{4x^2}{r^2}\sin^2\varphi \right) + \right.$$

$$\left. + \frac{3}{8}\left(\frac{x^4}{r^4} + \cancel{\frac{4x^3}{r^3}\sin\varphi} + \frac{4x^2}{r^2}\sin^2\varphi \right) \right\} =$$

$$= \frac{\gamma M_{\oplus}}{2r}\left\{ 2 - \frac{x^2}{r^2} + \frac{3}{4}\frac{x^4}{r^4} + 3\frac{x^2}{r^2}\sin^2\varphi \right\} =$$

$$= \frac{\gamma M_{\oplus}}{2r}\left\{ 2 + \frac{x^2}{r^2} \cdot (3\sin^2\varphi - 1) + \frac{3}{4} \cdot \frac{x^4}{r^4} \right\}$$

$$\Rightarrow V(r,\varphi) \approx \frac{\gamma M_\oplus}{r}\left(1 + \frac{x^2}{r^2} \cdot \frac{3\sin^2\varphi - 1}{2}\right)$$

$$\Rightarrow \frac{\gamma M_\oplus}{r}\left(1 + \frac{x^2}{r^2} \cdot \frac{3\sin^2\varphi - 1}{2}\right) = \frac{\gamma M_\oplus}{r}\left(1 + J_2\left(\frac{R_\oplus}{r}\right)^2 \cdot \frac{3\sin^2\varphi - 1}{2}\right)$$

$$\Rightarrow x^2 = J_2 R_\oplus^2$$

$$x = R_\oplus \sqrt{J_2} \cancel{= 6400\,km \cdot \sqrt{1{,}08 \cdot 10^{-1}}} \approx$$

$$\cancel{\approx 0{,}04 \cdot 6400 \approx 250\,km}$$

$$x = 6400 \cdot \sqrt{1{,}08 \cdot 10^{-3}} = 64\cancel{00} \cdot \sqrt{10{,}8} \cdot \cancel{10^{-2}} \approx$$

$$\approx 64 \cdot 3{,}3 \approx 210\,km$$

~~Задача 1~~

Чернова

$$\frac{3{,}6 \cdot 3{,}6}{\begin{array}{r} 216 \\ +\ 108 \\ \hline 12{,}96 \end{array}}$$

~~~~

$$\frac{13 \cdot 4}{52}$$

$$\frac{13 \cdot 5}{65}$$

$$\frac{6{,}4 \cdot 3{,}3}{\begin{array}{r} 192 \\ +\ 192 \\ \hline 21{,}12 \end{array}}$$

$$\frac{13 \cdot 6}{78}$$

$$\begin{array}{l} 75 : 13 \approx 5{,}8 \\ -65 \\ \hline 100 \end{array}$$

$$\begin{array}{r} 78 \\ +13 \\ \hline 91 \\ +13 \\ \hline 104 \end{array}$$

2. 1. двата ~~стари~~ гиганта
2. двете джуджета → по-стари

$$V(R_\oplus, 0) = \frac{\gamma M_\oplus}{R_\oplus}\left(1 - J_2 \cdot 1^2 \cdot \frac{0-1}{2}\right) =$$

$$= \frac{\gamma M_\oplus}{R_\oplus}\left(1 + \frac{J_2}{2}\right)$$

$$\frac{3{,}3 \cdot 3{,}3}{\begin{array}{r} 99 \\ +\ 99 \\ \hline 10{,}89 \end{array}}$$
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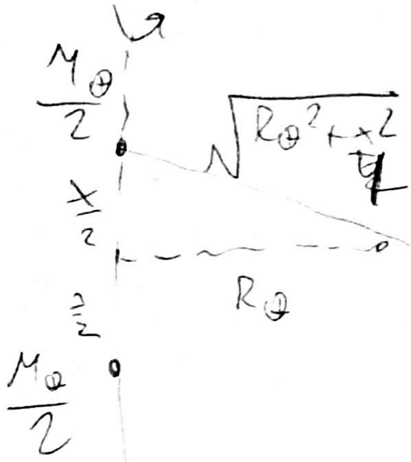


$$V(R_\oplus, 0) = 2 \cdot \frac{\gamma \frac{M_\oplus}{2}}{\sqrt{R_\oplus^2 + \frac{x^2}{4}}} =$$

$$= \frac{\gamma M_\oplus}{R_\oplus} \cdot \frac{1}{\sqrt{1 + \frac{x^2}{4R_\oplus^2}}}$$

$$\cancel{\approx \frac{\gamma M_\oplus}{R_\oplus}\left(1 - \frac{x^2}{8R_\oplus^2}\right)}$$

$$\frac{\gamma M_\oplus}{\cancel{R_\oplus}} \cdot \frac{1}{\sqrt{1 + \frac{x^2}{4R_\oplus^2}}} = \frac{\gamma M_\oplus}{\cancel{R_\oplus}} \cdot \left(1 + \frac{J_2}{2}\right)$$

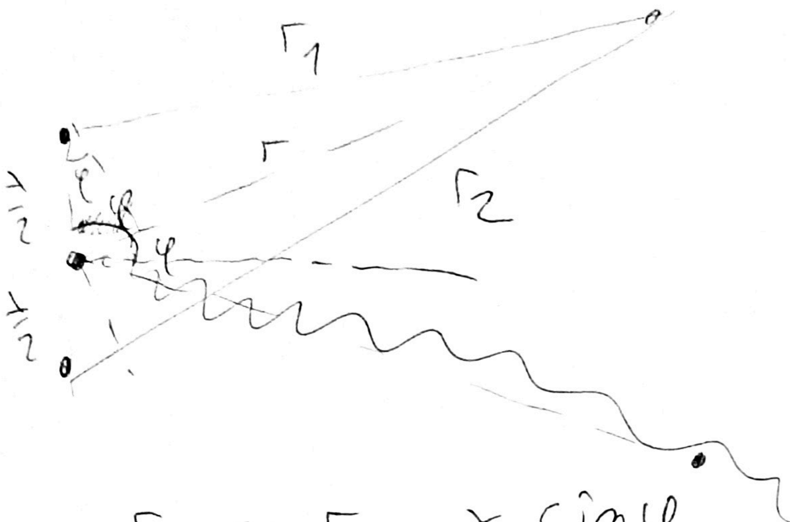
$$\varphi = 90^\circ : \quad V(R_\oplus, 90^\circ) = \frac{\gamma M_\oplus}{R_\oplus}\left(1 - J_2 \cdot 1^2 \cdot \frac{3-1}{2}\right) =$$

$$= \frac{\gamma M_\oplus}{R_\oplus}\left(1 - J_2\right)$$

$$V(R_\oplus, 90^\circ) = \frac{\gamma M_\oplus}{2\left(R_\oplus - \frac{x}{2}\right)} + \frac{\gamma M_\oplus}{2\left(R_\oplus + \frac{x}{2}\right)} =$$

$$= \frac{\gamma M_\oplus}{2R_\oplus} \cdot \left[\frac{1}{1 - \frac{x}{2R_\oplus}} + \frac{1}{1 + \frac{x}{2R_\oplus}}\right] \approx$$

$$\approx \frac{\gamma M_\oplus}{2R_\oplus} \cdot \left[1 + \frac{x}{2R_\oplus} + \right.$$



$$r_1 \approx r - \frac{x}{2}\sin\varphi$$

$$r_2 \approx r + \frac{x}{2}\sin\varphi$$

$$V(r,\varphi) = \frac{\gamma M_\oplus}{2r_1} + \frac{\gamma M_\oplus}{2r_2} =$$

$$= \frac{\gamma M_\oplus}{2\left(r - \frac{x}{2}\sin\varphi\right)} + \frac{\gamma M_\oplus}{2\left(r + \frac{x}{2}\sin\varphi\right)} =$$

$$= \frac{\gamma M_\oplus}{2r}\left\{\frac{1}{1-\frac{x}{2r}\sin\varphi} + \frac{1}{1+\frac{x}{2r}\sin\varphi}\right\}$$

$$f(x) = (1+x)^n \qquad f'(x) = n(1+x)^{n-1}$$

$$f''(x) = n(n-1)(1+x)^{n-2}$$

$$f'''(\cancel{x}) = n(n-1)(n-2)(1+x)^{n-3}$$

$$(1+x)^n \approx \frac{f(0)}{0!} + f'(0)\cdot\frac{(x-0)}{1!} + f''(0)\cdot\frac{(x-0)^2}{2!} + f'''(0)\cdot\frac{(x-0)^3}{3!}$$

$$= 1 + n \cdot 1 x + \frac{n(n-1) \cdot x^2}{2} + \frac{n(n-1)(n-2) x^3}{6}$$

$n = -1$:

$$(1+x)^{-1} = 1 - x + \frac{-1 \cdot (-2)}{2} \cdot x^2 + \frac{-1 \cdot (-2) \cdot (-3)}{6} x^3 =$$

$$= 1 - x + x^2 - x^3$$

$$\Rightarrow \frac{\gamma M_\oplus}{2r} \left(\frac{1}{1 - \frac{x}{2r} \sin\varphi} + \frac{1}{1 + \frac{x}{2r} \sin\varphi} \right) =$$

$$= \frac{\gamma M_\oplus}{2r} \Big[1 + \cancel{\frac{x}{2r} \sin\varphi} + \frac{x^2}{4r^2} \sin^2\varphi + \cancel{\frac{x^3}{8r^2} \sin^3\varphi} +$$

$$+ 1 - \cancel{\frac{x}{2r} \sin\varphi} + \frac{x^2}{4r^2} \sin^2\varphi - \cancel{\frac{x^3}{8r^3} \sin^3\varphi} \Big] =$$

$$= \frac{\gamma M_\oplus}{r} \left[1 + \frac{x^2}{4r^2} \sin^2\varphi \right]$$

$$\frac{\cancel{0{,}46}\ \cancel{0{,}23}\ 0{,}077}{\cancel{6}000} = \frac{7{,}7\cdot 10^{-2}}{10^{3}} = 7{,}7\cdot 10^{-5}$$

$\cancel{3}$

$$V = \sqrt{\frac{\gamma M}{R}} \qquad \frac{R^3}{T^2} = \frac{\gamma M}{4\pi^2}$$

$$R = \frac{\gamma M}{v^2}$$

$$\frac{\gamma^{\cancel{3}\,2} M^{\cancel{3}\,2}}{v^6 T^2} = \frac{\cancel{\gamma M}}{4\pi^2}$$

$$(\gamma M)^2 = \frac{v^3 T^2}{4\pi^2}$$

$$\gamma M = v^3 T$$

~~2310~~

$30 \cdot 7 = 210$

$30 \cdot 7{,}7 = 230$

$72 : 3 = 24$
-6
12

$$\frac{0{,}64 \cdot 0{,}8}{}$$

$64 \cdot 8$
512

$\sqrt{0{,}22} \simeq 0{,}45$

43
$+29$
72

$0{,}45 \cdot 1{,}4$
180
$+\ 45$
$0{,}630$

36
-22
14

$$\sqrt{x^2+r^2+2rx\cos\varphi} \approx \sqrt{r^2+2rx\cos\varphi}$$

$$\ldots\ldots = \sqrt{r^2-2rx\cos\varphi}$$

$$\frac{1}{\sqrt{r^2+2rx\cos\varphi}} + \frac{1}{\sqrt{r^2-2rx\cos\varphi}} \approx$$

$$\approx \frac{1}{r}\left(1-\cancel{\frac{1}{2}\cdot\frac{2x}{r}\cos\varphi} + \frac{3}{8}\cdot\left(\frac{2x}{r}\cos\varphi\right)^2 + \right.$$

$$\left. + 1 + \cancel{\frac{1}{2}\cdot\frac{2x}{r}\cos\varphi} + \frac{3}{8}\left(-\frac{2x}{r}\cos\varphi\right)^2\right) =$$

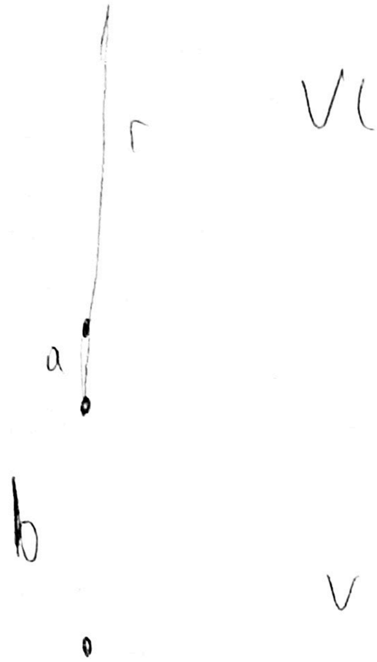
$$= \left(2 + \frac{3}{\cancel{4}}\cdot\frac{\cancel{4}x^2}{r^2}\cos^2\varphi\right) =$$

$$= 2 + \frac{3x^2}{r^2}\cdot(1-\sin^2\varphi) =$$

$$= 2\left(1 - \frac{x^2}{r^2}\cdot\frac{3\sin^2\varphi-3}{2}\right)$$

$$\frac{n(n-1)(n-2)}{6} = \frac{-\frac{1}{2}\cdot-\frac{3}{2}\cdot-\frac{5}{2}}{6} = \frac{-\cancel{15}\,5}{\cancel{6}\,2\cdot 8} = -\frac{5}{16}$$

$$-\frac{5}{16r}\cdot\frac{x^4}{r^4}$$



$$V(r, 90°) = \frac{\gamma M_\oplus}{2(r-a)} + \frac{\gamma M_\oplus}{2(r+b)} \approx$$

$$= \frac{\gamma M_\oplus}{2r}\left(1 + \frac{a}{r} + \frac{a^2}{r^2} + 1 - \frac{b}{r} + \frac{b^2}{r^2}\right) =$$

$$= \frac{\gamma M_\oplus}{2r}\left(1 + \frac{a-b}{r} + \frac{a^2+b^2}{r^2}\right)$$

$$r_2 = b\sin\varphi + r \, \cancel{\sin\varphi}$$

$$r_1 = -a\sin\varphi \; r \, \cancel{\sin\varphi}$$

$$V = \frac{\gamma M_\oplus}{2r_1} + \frac{\gamma M_\oplus}{2r_2} = \frac{\gamma M_\oplus}{2r}\left(\frac{1}{1-\frac{a}{r}\sin\varphi} + \frac{1}{1+\frac{b}{r}\sin\varphi}\right) =$$

$$\approx \frac{\gamma M_\oplus}{2r}\left(1 + \frac{a}{r}\sin\varphi + \frac{a^2}{r^2}\sin^2\varphi + 1 - \frac{b}{r}\sin\varphi + \right.$$

$$\left. + \frac{b^2}{r^2}\sin^2\varphi\right) \overset{a=b}{=} \frac{\gamma M_\oplus}{2r}\left(\right.$$

$$\frac{64 \cdot 4}{256}$$

$$\frac{-\frac{1}{2}\cdot\left(-\frac{1}{2}-1\right)}{2} = \frac{-\frac{1}{2}\cdot\frac{-3}{2}}{2}$$

$$\cos'\varphi = -\sin\varphi = 0$$

$$\cos\varphi \approx 1 - \frac{\varphi^2}{2}$$

$$\cos''\varphi = -\cos\varphi = -1$$

$$V(r,90^\circ) = \frac{\gamma M_\oplus}{2\left(r-\frac{d}{2}\right)} + \frac{\gamma M_\oplus}{2\left(r+\frac{d}{2}\right)} =$$

$$= \frac{\gamma M_\oplus}{2r}\left(\frac{1}{1-\frac{d}{2r}} + \frac{1}{1+\frac{d}{2r}}\right)$$

$$(1+x)^{-1} \simeq 1 - x + \frac{-1\cdot(-2)}{2}x^2 =$$

$$= 1 - x + x^2 - x^3$$

$$\rightarrow V = \frac{\gamma M_\oplus}{2r}\left(1 + \cancel{\frac{d}{2r}} + \frac{d^2}{4r^2} + 1 - \cancel{\frac{d}{2r}} + \frac{d^2}{4r^2} \pm \right.$$
$$\left. \pm \cancel{-\frac{d^3}{8r^3}} + \cancel{\frac{d^3}{8r^3}}\right)$$

~~[illegible]~~ ako

$$\frac{dV}{dr} = -\frac{\gamma M_\oplus}{r^2} + \frac{3\gamma M_\oplus}{r^4} J_2 R_\oplus^2 \cdot \frac{3\sin^2\varphi - 1}{2}$$

$$\varphi = \frac{\pi}{2}; \quad \frac{dV}{dr} = -\frac{\gamma M_\oplus}{r^2} + 3J_2\frac{\gamma M_\oplus R_\oplus^2}{r^4} = -g$$

$$g = \frac{\gamma M_\oplus}{}$$