$180 \cdot 60$

$180 \cdot 3600$
540
108000
648000

$\theta = 1{,}22 \frac{\lambda}{D}$

1. naloga

Kapela je tesno dvozvezdje, ki ga tvorita skoraj enaki zvezdi. Prvič so astronomi lahko komponenti brez uporabe interferometra zanesljivo razločili z vesoljskim teleskopom Hubble v ultravijolični svetlobi pri valovni dolžini 3000 Angstremov. Oceni kotno razdaljo med komponentama. Premer primarnega zrcala teleskopa Hubble je 2,4 metra

We state the equation, which links angular resolution θ in radians; wavelength λ in meters and aperture diameter in meters.

$$\left[\lambda = 3000\,\text{Å} = 3000\,\text{Å} \cdot \frac{1\,\text{m}}{10^{10}\,\text{Å}} = 3 \cdot \frac{10^3}{10^{10}}\,\text{m} = 3 \cdot 10^{-7}\,\text{m}\right]$$

$$\left[\theta = 1{,}22 \cdot \frac{\lambda}{D} = 1{,}22 \cdot \frac{3 \cdot 10^{-7}\,\cancel{\text{m}}}{2{,}4\,\cancel{\text{m}}} = 0{,}5 \cdot 3 \cdot 10^{-7} = \underline{\underline{1{,}5 \cdot 10^{-7}\,(\text{rad})}}\right]$$

$\frac{1{,}22}{2{,}4} \approx 0{,}5$

We can't easily imagine Radians, thus we need to convert them to angular seconds, or so.

$$\left[\theta = 1{,}5 \cdot 10^{-7}\,\text{rad} \cdot \frac{180^\circ}{\pi\,\text{rad}} = 1{,}5 \cdot 10^{-7}\,\cancel{\text{rad}} \cdot \frac{648000''}{\pi\,\cancel{\text{rad}}} = 0{,}5 \cdot 648000'' \cdot 10^{-7} = \right.$$

$$= 0{,}5 \cdot 6{,}5 \cdot 10^5 \cdot 10^{-7}\,'' = \underline{\underline{3{,}2 \cdot 10^{-2}\,''}} \; (= 32 \text{ marcs})$$

The result seems to be the right value (angular resolution of typical scope is around $\approx 1''$).

150000000 km

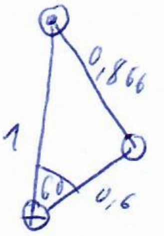
1,5 · 0,6

9,0

2. naloga

Asteroid s polmerom 50 metrov je bil v določenem trenutku oddaljen 0,866 a.e. od Sonca in ob opazovanju z Zemlje je bil kot med asteroidom in Soncem 60°. Oceni navidezno magnitudo asteroida v trenutku opazovanja. Ali ga je mogoče videti s teleskopom s 50-centimetrskim objektivom? Optične lastnosti površine asteroida so enake kot pri Luni.

Trigonometric functions are here hard to calculate, so let's make quick figure on the next page. It looks like this:



One can easily see that the angle $\sphericalangle \odot O \oplus$ is nearly equal to $90°$. That means, that the asteroid looks like this: 

We know, that Moon's magnitude is around -12^m on the full moon.

What is the magnitude of asteroid on full phase on the distance of 0,6 AU (look figure = measured data)?

$\left[0{,}6\,AU = 0{,}6\,\cancel{AU}\ \dfrac{1{,}5\cdot 10^8 km}{1\,\cancel{AU}} = 0{,}9\cdot 10^8 km = 9\cdot 10^7 km\right]$

$\Big[m_2 = m_1 - 5\log\dfrac{d}{9\cdot 10^7 km} = -12 - 5\cdot\log\dfrac{3{,}84\cdot 10^5 km}{9\cdot 10^7 km} = -12 - 5\cdot\log(4{,}2\cdot 10^{-3}) =$

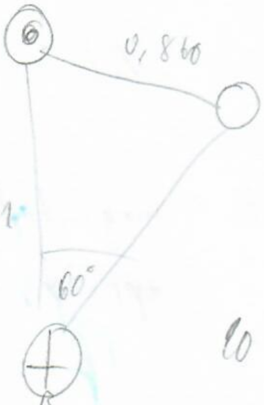
$= -12 - 5(\log 4{,}2 + \log 10^{-3}) = -12 - 5(0{,}62 - 3) = 0^m\Big]$

An object like Moon on this distance would have magnitude of 0^m (full phase).

2nd problem

$\frac{0,866 \cdot 5}{4,330}$

$m_2 - m_1 = -2,5 \log\left(\frac{b_2}{b_1}\right)$



$10^{0,4(m_1-m_2)} = \frac{b_2}{b_1}$

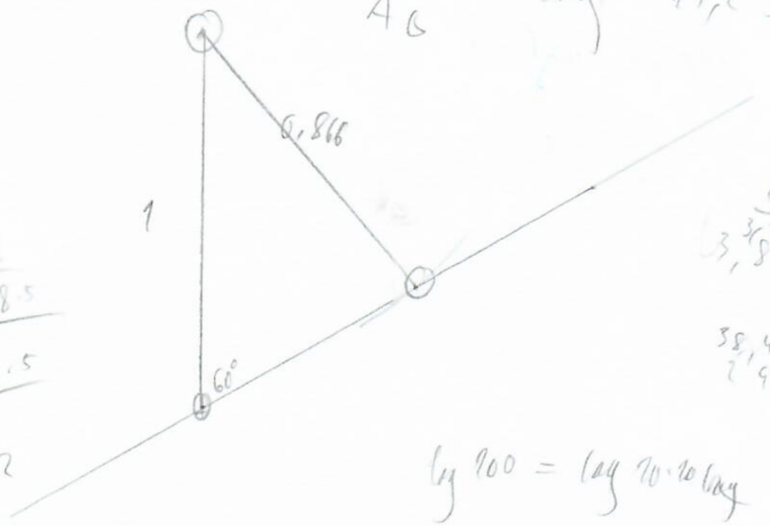
$m_2 - m_1 = +2,5 \log 4,5$

$m_2 - m_1 = 2,5 \cdot 0,7$

$\frac{B_☾}{A_☾} = \frac{B_0}{A_0}$

$3,2 : 5 = 0,6\ AU$

$B_0 = \frac{A_0 B_☾}{A_☾}$



$\log 100 = \log 10 \cdot 10$

How much bigger is Moon's cross-section than asteroid's?

$$\left[\frac{A_☾}{A_0} = \frac{2\pi r_☾}{2\pi r_0} = \frac{r_☾}{r_0} = \cancel{1700}\ \frac{1{,}7 \cdot 10^6}{5 \cdot 10} = \underline{3{,}4 \cdot 10^4}\right]$$

~~Brightness ∝~~

$$B \propto A \Longrightarrow \left[m_0 - m_☾ = -2{,}5 \log \frac{A_0}{A_☾} = 2{,}5 \log \frac{A_☾}{A_0} = \right.$$

$$\left. = 2{,}5(\log 10^4 + \log 3{,}4) = 2{,}5(4 + 0{,}5) = 11{,}3\right]$$

$m_0 = 11,3 + m_☾ = \underline{11,3}$ → this is the real asteroid's magnitude on the full phase.

But the asteroid is half full. 2nd problem

For every magnitude, brightness falls for 2,5.

For brightness fall of 2, the magnitude falls approx. 0,8.

$$\left[11,3 + 0,8 = 12,1 \approx \boxed{\underline{12}^{m}} \right]$$

we got a lot of approximations

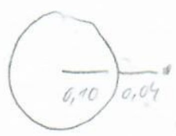
$1{,}5 \cdot 1{,}5$

$2{,}25 \cdot 1{,}5$

$3{,}4$

$0{,}14 \cdot 1{,}5 \cdot 10^{11}$

$1{,}5$



$0{,}14 \cdot 0{,}14$

$0{,}0196 \cdot 0{,}14$

$0{,}02 \cdot 0{,}14$

$0{,}028$

$0{,}0028$

3. naloga

Dvozvezdje sestavljata glavna zvezda z največjim polmerom 0,10 a.e. in bela pritlikavka, ki se nahaja na razdalji 0,14 a.e. od središča glavne zvezde. Masa bele pritlikavke je enaka masi Sonca, z glavne zvezde se na pritlikavko z majhno hitrostjo steka snov v obliki akrecijskega diska. Oceni povprečno gostoto glavne zvezde v sistemu.

~~Volume of the first star: $0{,}10\,AU = 1{,}5 \cdot 10^{7}\,m$~~

~~$V = \frac{4}{3}\pi r^3 = \frac{4 \cdot 3 \cdot (1{,}5 \cdot 10^7)^3}{3} = 4 \cdot 3{,}4 \cdot 10^{21}\,m^3 = 14 \cdot 10^{21}\,m^3$~~

Equation for Roche limit:

Around 1000 km.

$$d = R_M \sqrt[3]{2\frac{\rho_M}{\rho_m}}$$

$$\Downarrow$$

$$\left[\rho_M = \frac{d^3 \cdot \rho_m}{2R_M^3} = \frac{d^3 \cdot M_m}{2R_M^3 \frac{4\pi}{3} R_m^3} = \right.$$

$$= \frac{(0{,}14 \cdot \cancel{1{,}5 \cdot 10^{11}})^3 \cdot 2 \cdot 10^{30}\,kg}{2 \cdot (0{,}10 \cdot \cancel{1{,}5 \cdot 10^{11}})^3 \cdot \frac{4}{3} \cdot 3{,}1 \cdot 10^6}\,\frac{kg}{m^3} =$$

$$= \frac{0{,}0028\,\cancel{2} \cdot 10^{30} \cdot 10^{-6}}{\cancel{2} \cdot 0{,}001 \cdot \frac{4}{3} \cdot 3{,}1}\,\frac{kg}{m^3} = \frac{2{,}8 \cdot \cancel{10^{-3}} \cdot 10^{30} \cdot 10^{-6}}{\frac{4}{3} \cdot \cancel{3{,}1} \cdot \cancel{10^{-3}}}\,\frac{kg}{m^3} =$$

$$\left. \frac{2{,}8}{4} \cdot 10^{24}\,\frac{kg}{m^3} = \boxed{6 \cdot 10^{23}\,\frac{kg}{m^3}} \right]$$

3^{rd} problem

$$\frac{0{,}5}{1000} \cdot 300\,000 \qquad \frac{0.5 \cdot 3 \cdot}{1} \; 10^5 \cdot 10^{-3}$$

$$\frac{1{,}5}{1} \qquad 10^2$$

4. naloga

Opazovanja dvozvezdja, sestavljenega iz nevtronske zvezde z maso 1,4 mase Sonca in zvezde glavne veje, so razkrila rentgenske pulzacije s povprečno periodo 1 sekunde, ki odstopajo od nje za največ 10^{-4} sekunde. Hkrati so spektralna opazovanja v optičnem območju pokazala, da se periodično spreminja tudi valovna dolžina spektralne črte Hα, pri čemer so odstopanja od srednje vrednosti za največ 0,5 Angstrema. Oceni izsev tega sistema v optičnem območju elektromagnetnega spektra.

What is the velocity of one star relative to another?

$$\left[v = c\left(1 - \frac{\lambda_0}{\lambda_c}\right) = c\left(\frac{\lambda_c - \lambda_0}{\lambda_c}\right) = \frac{0{,}5\,\text{Å}}{7000\,\text{Å}} \cdot c = \right.$$

$$\left. = \frac{0{,}5 \cdot 3}{7} \cdot 10^2 \frac{km}{s} = \underline{20\ \frac{km}{s}}\right]$$

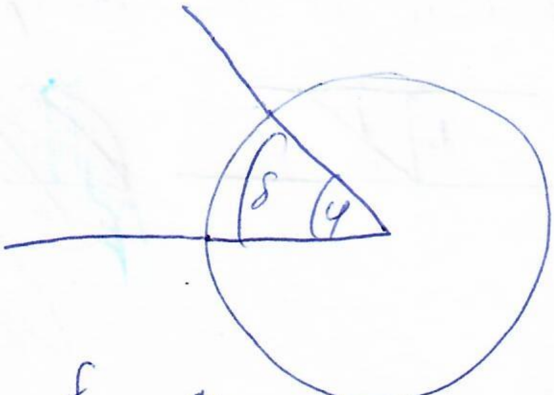
$$[s = v \cdot t = 20\,km]$$

$$\left[r = \frac{s}{2\pi} = \frac{20\,km}{2\pi} = 3\,km\right]$$

If we suppose, that the polar beams of neutron star don't point towards us, then most of the light comes from the second star.
The second star isn't so bright because $r = 3\,km$.
Sun's brightness is around 10^{27} W, so I would say that the brightness of the system is around $\boxed{10^8\,W}$.

5. naloga

Zvezda λ Zmaja ima koordinate $\delta = 69^\circ 20'$ in $\alpha = 11^h 31^m$. Njena navidezna magnituda iznad ozračja je 3,8. Kako je navidezna magnituda te zvezde odvisna od časovnega kota pri opazovanju v Murmansku? Zemljepisna širina Murmanska je $\phi = 68^\circ 58'$.



φ and δ are ~~bats~~ basically the same ⇒ λ Draconis is Vega like star → circumpolar and culminates in zenith.

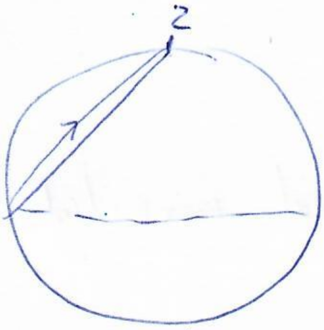


Fig 1

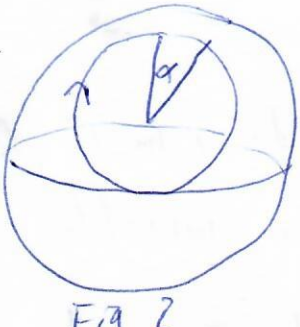
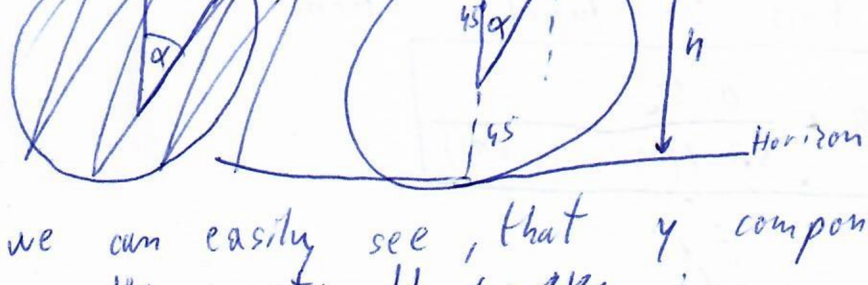


Fig 2

α = hour angle

Let's take a look at Fig. 2. We can approximate the angular height and spherical geometry to Euclidian one. We have triangle



And we can easily see, that y component is equal to $45 \cos \alpha$. We just add 45° to get the angular height:

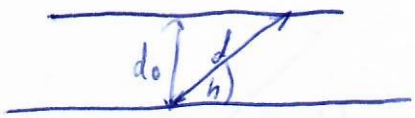
$[h = 45 \cos\alpha + 45 = 45(\cos\alpha + 1)]$ **5th problem**

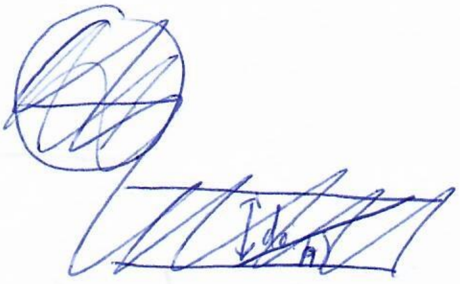
What is ~~extin~~ extinction ε ?

It changes the brightness b.

Let be d the thickness of the atmosphere.

Then:





$$d = \frac{d_0}{\sin h}$$

$$\varepsilon \propto d$$

Let be k_ε the factor, so that

$\left[\varepsilon = \cancel{d} \; \frac{d}{d_0} \cdot k_\varepsilon\right]$ is true. $\rightarrow$ I know from my experiences, that k_ε is around 0^m5.

The combined magnitude is:

$$[M = m + \varepsilon]$$

M is combined magnitude, m is ~~ap~~ real magnitude

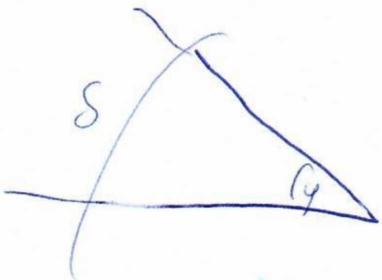
Let's only plug in the variables:

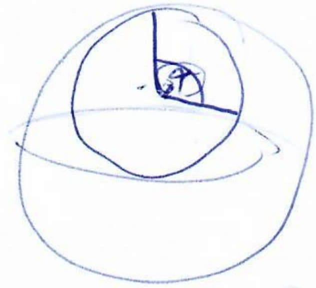
$$\left[M = m + \varepsilon = m + \frac{d}{d_0} k_\varepsilon = m + \frac{d_0}{d_0 \sin h} \cdot k_\varepsilon = m + \frac{k_\varepsilon}{\sin(45(\cos\alpha + 1))} = 3{,}8 + \frac{0{,}5}{\sin(45(\cos\alpha + 1))}\right]$$

Let's make this in function form:

$$\boxed{M(\alpha) = 3{,}8 + \frac{0{,}5}{\sin(45(\cos\alpha + 1))}}$$

5th problem





$\cos C =$ $90-\varphi$

$\cos c = \cos a \cos b + \sin a \sin b \cos C$ $45 \cos + 45$

$\frac{d_0}{d} = \sin \varphi$

$l_0 = \frac{d_0}{\sin\varphi}$

$d = \frac{1}{\sin\varphi} d_0$

$\varepsilon = l d$

$\varepsilon = 0{,}5 \cdot \frac{d}{d_0}$

$d = l_\varepsilon \cdot d_0$

$0{,}5 = d_0 \cdot l_\varepsilon$