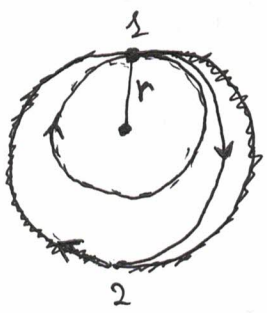
СамГУ - 11

1

№1

1) Рассмотрим схему первого варианта:



первый импульс:

$V_1 = V_0 + 0{,}1V_0 = 1{,}1V_0 \rightarrow \Delta V_1 = 0{,}1V_0$

$r_1 = r \cdot \frac{1}{1-e_1}$ $\longrightarrow$

$V_1^2 = V_{1cp}^2 \cdot \frac{1+e_1}{1-e_1}$

$V_1^2 = \frac{GM}{r_1} \cdot \frac{1+e_1}{1-e_1}$

$V_0^2 = \frac{GM}{r}$

$\longrightarrow \frac{GM}{r_1} \cdot \frac{1+e_1}{1-e_1} = \frac{GM}{r} \cdot 1{,}1^2$

$r_1 = \frac{1+e_1}{1-e_1} \cdot r \cdot \frac{1}{1{,}1^2} = r_1 \cdot \frac{(1+e_1)}{1{,}1^2} \rightarrow 1+e_1 = 1{,}21$

$e_1 = 0{,}21$

второй импульс:

$V_2 = V_1 - 0{,}1V_1 = 0{,}9V_1 = \rightarrow \Delta V_2 = -0{,}1V_1 = -0{,}11V_0$

$r_2 = r_1 \frac{1}{1+e_1}$

$V_2^2 = V_{2cp}^2 \cdot \frac{1-e_2}{1+e_2}$ $\longrightarrow$ $\frac{GM}{r_2} \cdot \frac{1-e_2}{1+e_2} = \frac{GM}{r_1} \cdot 0{,}9^2$

$V_2^2 = \frac{GM}{r_2} \cdot \frac{1-e_2}{1+e_2}$

$V_1^2 = \frac{GM}{r_1}$

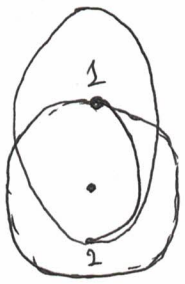
$r_2 = \frac{1-e_2}{1+e_2} r_1 \cdot \frac{1}{0{,}9^2} = \frac{(1-e_2)}{0{,}9^2} \cdot r_2 \rightarrow$

$\rightarrow 1-e_2 = 0{,}81$

$e_2 = 0{,}19$

$r_2 = r_1 \cdot \frac{1}{1+e_2} = r \cdot \frac{1}{1+e_2} \cdot \frac{1}{1-e_1}$

2) Рассмотрим вторую схему:



первый импульс:

$$\begin{cases} V_1' = V_0 + \Delta V_2 = V_0 - 0{,}11 V_0 = 0{,}89 V_0 \\ r_1' = \dfrac{r}{1+e_1'} \\ V_1'^2 = \dfrac{GM}{r_1'} \cdot \dfrac{1-e_1'}{1+e_1'} \\ V_0^2 = \dfrac{GM}{r} \end{cases} \longrightarrow$$

$$\longrightarrow \frac{GM}{r_1'} \cdot \frac{1-e_1'}{1+e_1'} = 0{,}89^2 \cdot \frac{GM}{r} \longrightarrow r_1' = \frac{r}{1+e_1'} \cdot \frac{(1-e_1')}{0{,}89^2} = r_1' \cdot \frac{(1-e_1')}{0{,}89^2}$$

$$1 - e_1' = 0{,}7921$$

$$e_1' = 0{,}2079$$

Второй импульс:

$$\begin{cases} V_2' = V_1' + \Delta V_1 = V_0 \cdot 0{,}89 + 0{,}1 V_0 = 0{,}99 V_0 = \dfrac{0{,}99}{0{,}89} V_1' \\ r_2' = \dfrac{r}{1-e_2'} \\ V_2'^2 = \dfrac{GM}{r_2'} \cdot \dfrac{1+e_2'}{1-e_2'} \longrightarrow \dfrac{GM}{r_2'} \cdot \dfrac{1+e_2'}{1-e_2'} = \dfrac{0{,}99^2}{0{,}89^2} \cdot \dfrac{GM}{r_1'} \longrightarrow \\ V_2^2 = \dfrac{GM}{r_1} \end{cases}$$

$$\longrightarrow r_2' = \frac{1+e_2'}{1-e_2'} \cdot \frac{r_1'}{\left(\frac{0{,}99^2}{0{,}89}\right)} = \frac{(1+e_2')}{\frac{0{,}99^2}{0{,}89^2}} \cdot r_1' \longrightarrow 1+e_2' = \frac{0{,}99^2}{0{,}89^2}$$

$$e_2' = \left(\frac{99}{89}\right)^2 - 1$$

3) $$r_2' = \frac{r}{(1-e_2')(1+e_1')} \longrightarrow T_2' = T \cdot \sqrt{\left(\frac{1}{(1-e_2')(1+e_1')}\right)^3}$$

$$r_2'' = \frac{r}{(1+e_2')(1-e_1)} \longrightarrow T_2'' = T \cdot \sqrt{\left(\frac{1}{(1+e_2)(1-e_1)}\right)^3}$$

$$\Delta T = 1\text{сут} \cdot \left| \left(\frac{1}{(1-e_2')(1+e_1')}\right)^{\frac{3}{2}} - \left(\frac{1}{(1+e_2)(1+e_1)}\right)^{\frac{3}{2}} \right|$$

№2

(3)

1) $S = \alpha_\odot + t_\odot = T_\odot - 12^h + \alpha_\odot = \alpha_\odot - 12^h$

$\alpha_\odot = 18^h + \frac{9}{365} \cdot 24^h \approx 18^h 40^m$

(день зимнего солнцест.)

$S \approx 6^h 40^m$

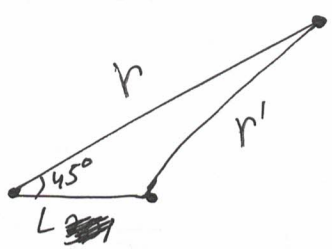
~~$\cos(t_*) = \sin(h) - \sin\varphi \sin\delta$~~

$\cos\varphi \cdot \cos\delta$

$t_* = S - \alpha_* \approx 6^h 45^m - 6^h 40^m \approx 0 \Rightarrow$ верхняя кульминация

$h_{в.к.} = 90° - \varphi + \delta = 90° - 28° - 17° = 45°$

2)



$L = v \cdot t = 30 \frac{км}{с} \cdot 1 с = 30 м$

$r'^2 = r^2 + L^2 - 2 \cdot r \cdot L \cdot \cos 45° \; | : r^2$

$\left(\frac{r'}{r}\right)^2 = 1 + \frac{L^2}{r^2} - 2 \cdot \left(\frac{L}{r}\right) \cdot \frac{\sqrt{2}}{2}$

3) $m = M - 5 + 5\lg(r)$

$m' = M - 5 + 5\lg(r')$

$\Delta m = m' - m = 5\lg\left(\frac{r'}{r}\right)$

$\Delta m = 2{,}5 \lg\left(\frac{r'^2}{r^2}\right)$

$\Delta m = 2{,}5 \cdot 10^{-16} \cdot \frac{1{,}4}{2{,}3} \approx 1{,}5 \cdot 10^{-16} м$

$r \sim 10$ пк :

$\frac{L}{r} = \frac{30 м}{10 \cdot 206265 \cdot 1{,}5 \cdot 10^{11} м} \approx \frac{3}{10^5 \cdot 2 \cdot 1{,}5 \cdot 10^{11}} \approx$

$\approx 10^{-16}$

$\left(\frac{r'}{r}\right)^2 = 1 + (10^{-16})^2 - \sqrt{2} \cdot 10^{-16} \approx$

$\approx 1{,}4 \cdot 10^{-16} + 1$

$\lg\left(\left(\frac{r'}{r}\right)^2\right) = \frac{\ln\left(\left(\frac{r'}{r}\right)^2\right)}{\ln(10)} \approx \frac{1{,}4 \cdot 10^{-16}}{2{,}3}$

Ответ: $\Delta m = 1{,}5^m \cdot 10^{-16}$

САМ-11

(4)

№3.

$M = 2M_\odot$

$T = 4$ года

1) $L \sim M^4 \rightarrow \frac{L}{L_\odot} = \frac{M^4}{M_\odot^4} \rightarrow L = 16 L_\odot$

2) $M \cdot T^2 = a^3 \rightarrow a^3 = 4^2 \cdot 2 \Rightarrow a = \sqrt[3]{32} = \sqrt[3]{2^5} = \sqrt[3]{1024^{\frac{1}{2}}} \approx$

$\approx \sqrt[3]{1000^{\frac{1}{2}}} = \sqrt[2]{10} \approx 3$ а.е.

3) $E_* = E_\odot \cdot \frac{L}{L_\odot} \cdot \left(\frac{a_\oplus}{a}\right)^2 = E_\odot \cdot \frac{16}{10} \approx 1{,}6\, E_\odot$, где $E_\odot = 1360 \frac{Вт}{м^2}$

4)

$E_p = E_* \cdot \overline{\cos i}$

$c = \int_0^{\frac{\pi}{2}} \cos i\, di = \cos(0) - \cos\left(\frac{\pi}{2}\right) = 1 \rightarrow \overline{\cos i} = \frac{c}{\left(\frac{\pi}{2} - 0\right)} = \frac{2}{\pi} \approx \frac{2}{3}$

$W = E_p \cdot 2 \cdot S \cdot \frac{t_{ур}}{2} \eta = 2 \cdot 1{,}6 \cdot \frac{2}{3} \cdot 100 м^2 \cdot 1360 \frac{Вт}{м^2} \cdot 10^4 \cdot 0{,}1$

$\approx 2 \cdot 100 м^2 \cdot 1360 \frac{Вт}{м^2} = 662$ кДж

Ответ: $W = 662$ кДж.

сам-11

5

№4.

1) $m = -5 + 5\lg(r) + M + (A \cdot r)$

$5{,}7 = (A \cdot r) + 5$

$(A \cdot r) = 0{,}7$

звезда лежит за туманностью

$\lg(r) = \lg(3{,}1) + 2$

$\lg(3{,}1) \approx \lg(1 + 0{,}7) \approx$

$\approx \frac{1}{2{,}3} \cdot \left(0{,}7^2 + \frac{0{,}7^2}{2} + \frac{0{,}7^3}{3} + \frac{0{,}7^4}{4}\right) \approx$

$\approx 1 - \frac{1{,}5}{2{,}3} \approx -\frac{1}{2} + 1 =$

$= 0{,}5$

$\lg(r) = 2 + 0{,}5 = 2{,}5$

$\frac{L'_*}{L_*} = \frac{L_* - L_T}{L_*} = 1 - \frac{L_T}{L_*} = 10^{-0{,}4 \cdot A \cdot r} \approx 10^{-0{,}2} \approx \frac{1}{\sqrt{2{,}512}} \approx \frac{1}{1{,}5}$

$1 - \frac{L_T}{L_*} = \frac{2}{3} \rightarrow L_T = \frac{1}{3} L_*$

$\frac{L_* - L_T}{r^2} = \frac{L_T}{r_T^2}$

$\frac{\left(1 - \frac{1}{3}\right) L_*}{r^2} \quad \frac{\frac{1}{3} L_*}{r_T^2} \rightarrow$

$r_T^2 = \frac{1}{2} r^2 = 370^2 \cdot \frac{1}{2}$

$r_T = \sqrt{\frac{370^2}{2}} \approx 270\,пк$

$\Delta r = r - r_T = 370\,пк - 270\,пк =$

$= 100\,пк$

Ответ: $\Delta r = 100\,пк$.