XXVIII Санкт-Петербургска олимпиада
по астрономия
Теоретичен тур
31.01.2021 г.

Задача 1. $r_0 \simeq 42200\,km$ - геостационарна орбита
$T_0 \simeq 24^h$

Планирам трансфер:

$V_0 = \frac{2\pi r_0}{T_0} \simeq 3{,}1\,km/s$

$V_1 = V_0 + 10\% V_0 \simeq 3{,}4\,km/s$

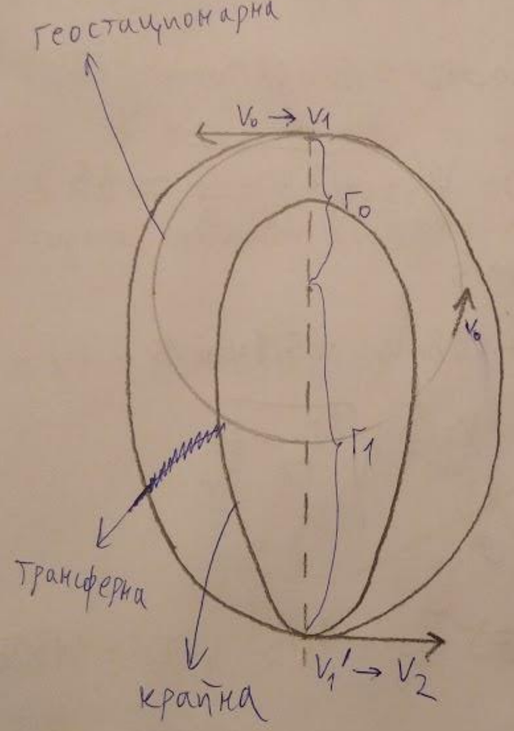
$$\left| \begin{array}{l} V_1 = \sqrt{\gamma M\left(\frac{2}{r_0} - \frac{1}{a}\right)} \\ V_0 = \sqrt{\frac{\gamma M}{r_0}} \end{array} \right. \quad /:$$

$\Rightarrow \frac{V_1}{V_0} = \sqrt{2 - \frac{r_0}{a}}$

$\frac{r_0}{a} = 2 - \frac{V_1^2}{V_0^2} \simeq 0{,}8$

$\Rightarrow \frac{a}{r_0} \simeq 1{,}25$

$\Rightarrow a \simeq 52\,800\,km$
(голяма полуос на трансферната орбита)



$a = \frac{r_0 + r_1}{2} \Rightarrow r_1 = 2a - r_0 = 63\,300\,km$

$V_1' = \sqrt{\gamma M\left(\frac{2}{r_1} - \frac{1}{a}\right)} \Rightarrow \frac{V_1'}{V_0} = \sqrt{\frac{2r_0}{r_1} - \frac{r_0}{a}} \simeq 0{,}72$

$\Rightarrow V_1' \simeq 2{,}2\,km/s \qquad V_2 = V_1' - 10\% V_1' \simeq 2{,}0\,km/s$

$$V_2 = \sqrt{\gamma M\left(\frac{2}{r_1} - \frac{1}{a_f}\right)} \Rightarrow \frac{V_2}{V_0} = \sqrt{\frac{2r_0}{r_1} - \frac{r_0}{a_f}}$$

$$\Rightarrow \frac{r_0}{a_f} = \frac{2r_0}{r_1} - \frac{V_2^2}{V_0^2} \approx 0{,}9 \Rightarrow \frac{a_f}{r_0} = 1{,}1$$ → голяма полуос на крайната орбита

$$\left(\frac{T_f}{T_0}\right)^2 = \left(\frac{a_f}{r_0}\right)^3 \Rightarrow T_f = T_0 \cdot \left(\frac{a_f}{r_0}\right)^{\frac{3}{2}} \approx 1{,}15\,T_0$$

Проведен трансфер:

$$V_1 = V_0 - 10\%\,V_0 \approx 2{,}8\,km/s$$

$$V_1 = \sqrt{\gamma M\left(\frac{2}{r_0} - \frac{1}{a}\right)} \Rightarrow \frac{V_1}{V_0} = \sqrt{2 - \frac{r_0}{a}}$$

$$\frac{r_0}{a} = 2 - \left(\frac{V_1}{V_0}\right)^2 \approx 1{,}2 \Rightarrow a \approx 0{,}8\,r_0 \approx 33800\,km$$

$$a = \frac{r_0 + r_1}{2} \Rightarrow r_1 = 2a - r_0 \approx 25300\,km$$

$$V_1' = \sqrt{\gamma M\left(\frac{2}{r_1} - \frac{1}{a}\right)} \Rightarrow \frac{V_1'}{V_0} = \sqrt{\frac{2r_0}{r_1} - \frac{r_0}{a}} \approx 1{,}5$$

$$\Rightarrow V_1' \approx 4{,}6\,km/s$$

$$V_2 = V_1' + 10\%\,V_1' \approx 5{,}1\,km/s$$

$$V_2 = \sqrt{\gamma M\left(\frac{2}{r_1} - \frac{1}{a_f}\right)} \Rightarrow \frac{V_2}{V_0} = \sqrt{\frac{2r_0}{r_1} - \frac{r_0}{a_f}}$$

$$\frac{r_0}{a_f} = \frac{2r_0}{r_1} - \left(\frac{V_2}{V_0}\right)^2 \approx 0{,}9$$

$$\Rightarrow \frac{a_f}{r_0} \approx 1{,}1 \Rightarrow \frac{T_f}{T_0} \approx 1{,}15, \quad T_f \approx 1{,}15\,T_0$$

⇒ Двете орбити имат практически еднакъв период

Задача 2.

$t_\odot$ - слънчево време s - звездно

На зимното слънцестояние (около 21.12)

$(s-t_\odot)_0 = 6^h$. Между 21.12 и 1.01 има приблизително 10 дена и ще се натрупа допълнителна разлика $\Delta(s-t_\odot) = 10 \cdot 3^m 56^s \approx 40^m$

На 1.01:

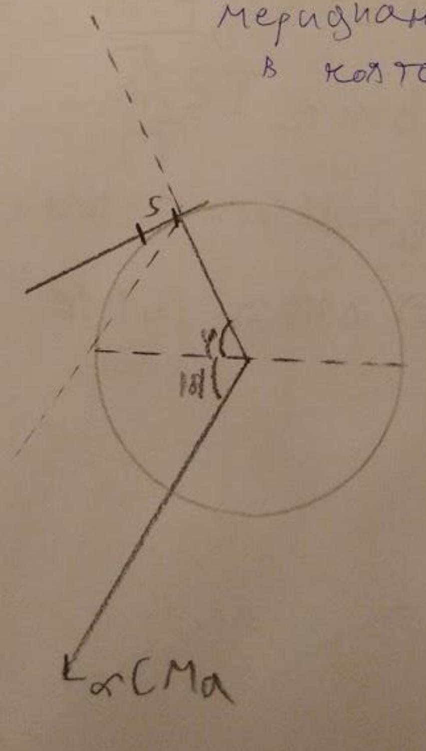
$$s - t_\odot = (s-t_\odot)_0 + \Delta(s-t_\odot) \approx 6^h 40^m$$

Действието протича в $t_\odot \approx 0^h$

$$s = t_\odot + (s - t_\odot) \approx 6^h 40^m$$

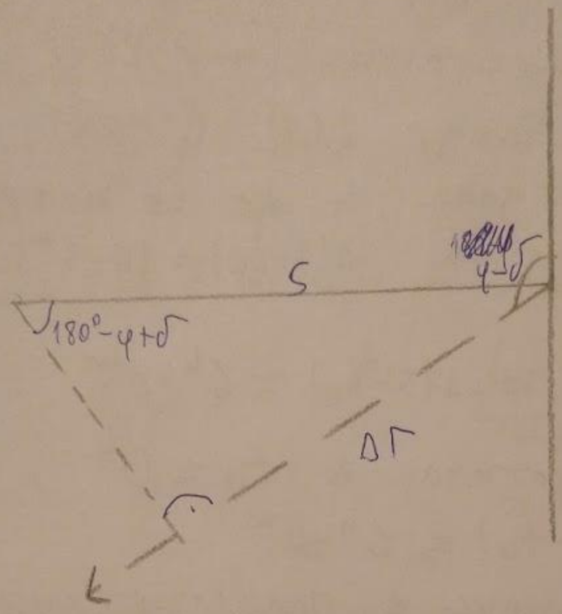
$s \approx \alpha \Rightarrow$ Сириус е приблизително в горна кулминация и меридианът на наблюдателите практически съвпада с меридиана на точката, в която Сириус е в зенита.

$s \approx vt = 30m$



$\Delta r = s \cdot \sin(180° - \varphi + \delta) = s \cdot \sin(\varphi - \delta) = s \cdot \sin 45°$

$\Delta r \approx 21 m$



$m = M - 5 + 5 \lg r$

$\Rightarrow \Delta m = 5 \lg \frac{r - \Delta r}{r} = 5 \lg\left(1 - \frac{\Delta r}{r}\right)$

$\Rightarrow \Delta m \approx -5 \frac{\Delta r}{r}$

$r \approx 4\, ly \approx 1{,}2\, pc \approx 2{,}4 \cdot 10^5\, AU \approx 3{,}6 \cdot 10^{19}\, m$

$\Rightarrow \Delta m \approx -3{,}1 \cdot 10^{-10}\, m$

Задача 3.

$$\frac{r_{AU}^3}{T_{yr}^2} = \frac{M}{M_\odot} \qquad r = \sqrt[3]{\frac{M}{M_\odot} T^2} \approx 3{,}2\ AU$$

$$\frac{L}{L_\odot} \approx \left(\frac{M}{M_\odot}\right)^4 = 16$$

$A_\odot = 1367\ W/m^2$ – слънчева константа

$$A = \frac{L}{4\pi r^2} \qquad A_\odot = \frac{L_\odot}{4\pi r_\oplus^2}$$

$$\Rightarrow \frac{A}{A_\odot} = \frac{L}{L_\odot} \cdot \frac{1}{r_{AU}^2} \approx 1{,}6$$

$$\Rightarrow A \approx 2200\ W/m^2$$

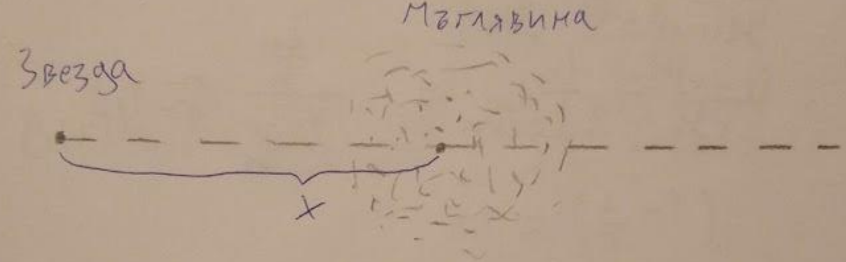
$$P = A \cdot S \qquad P_{eff} = \eta P = \eta A S$$

$$E = P_{eff} T = \eta A S T \approx 1{,}6 \cdot 10^9\ J$$

$(\eta = 10\%,\ S = 100 m^2,\ T = 20 h)$

Задача 4.

Нека приемем, че звездата е по-близо (ако не е така ще получим отрицателен резултат за разстоянието. Звездата може да е вътре в мъглявината, а може да бъде и извън нея (ако е по-близо задължително трябва да бъде вътре в нея ако има промяна в блясъка)



Очакваната ~~промяна в~~ видима звездна величина е:

$$m_0 = M - 5 + 5\lg r \approx 4^m,3 < 5^m,7$$

$\Rightarrow$ Със сигурност част от светлината е погълната от мъглявината.

L_n - светимост на мъглявината

L_s - на звездата

$$E_n = \frac{L_n}{4\pi(r-x)^2} \qquad E_s = \frac{L_s - L_n}{4\pi r^2}$$

$$m - M = -2{,}5\lg\left(\frac{E_s}{E_s(10pc)}\right) =$$

$$= -2{,}5\lg\left\{\frac{\frac{L_s - L_n}{4\pi r^2}}{\frac{L_s}{4\pi\cdot(10pc)^2}}\right\} = 5\lg\frac{r}{10pc} - 2{,}5\lg\left(1 - \frac{L_n}{L_s}\right)$$

$$8{,}4 = 7 - 2{,}5\lg\left(1 - \frac{L_n}{L_s}\right)$$

$$\Rightarrow \lg\left(1-\frac{L_n}{L_s}\right) \approx -0{,}56$$

$$1-\frac{L_n}{L_s} = 10^{-0{,}56} \approx 0{,}35$$

$$\frac{L_n}{L_s} \approx 0{,}65$$

$$E_n = E_s$$

$$\frac{L_n}{\cancel{4\pi}(r-x)^2} = \frac{L_s - L_n}{\cancel{4\pi} r^2}$$

$$\frac{(r-x)^2}{r^2} = \frac{L_n}{L_s - L_n} = \frac{\frac{L_n}{L_s}}{1-\frac{L_n}{L_s}} \approx 1{,}8$$

$$\left(1-\frac{x}{r}\right)^2 \approx 1{,}8$$

$$\Rightarrow \frac{x}{r} \approx 0{,}05$$

$$\Rightarrow x \approx 1{,}55\,pc$$

$\frac{x}{r} > 0 \Rightarrow$ Звездата е по-далеч, мъглявината по-близо

Задача 5.

$\varepsilon = 10 keV$ $\qquad \varepsilon = \hbar\omega \Rightarrow \omega = \frac{\varepsilon}{\hbar}$

На повърхността:

$$F = evB_0 = m_e\frac{v^2}{R} \Rightarrow eB_0 = m_e \cdot \frac{v}{R}$$

$$\frac{v}{R} = \omega = \frac{\varepsilon}{\hbar} \Rightarrow eB_0 = m_e\frac{\varepsilon}{\hbar}$$

$$B_0 = \frac{m_e\varepsilon}{e\hbar}$$

$$B(r) = B_0 \cdot \frac{R^3}{r^3} = \frac{m_e \varepsilon R^3}{e\hbar} \cdot \frac{1}{r^3}$$

Разглеждаме границата на магнитосферата като слой от N протони, падащ със скорост v и приемаме, че светимостта на звездата се дължи на загубата на орбитална енергия от този слой:

$$L = \frac{dE_{орб}}{dt} = \frac{d}{dt}\left(-\frac{\gamma mM}{2r}\right) = \frac{\gamma mM}{2r^2}\frac{dr}{dt} = \frac{\gamma mM}{2r^2}v$$

$$\Rightarrow v = \frac{2r^2L}{\gamma mM}, \quad m = Nm_p$$

$$F = NevB = \cancel{N}e \cdot \frac{2r^2L}{\gamma \cancel{N} m_p M}B = \frac{2r^2Le}{\gamma m_p M}B$$

$$p = \frac{F}{4\pi r^2} = \frac{Le}{2\pi\gamma m_p M}B$$

$$p = kB^2$$

$$\Rightarrow \frac{Le}{2\pi\gamma m_p M}\cancel{B} = \kappa B^{\cancel{2}}$$

$$\frac{Le}{2\pi\gamma m_p M} = \kappa \cdot \frac{m_e \varepsilon}{e\hbar} \cdot \left(\frac{R}{r}\right)^3$$

$$\left(\frac{r}{R}\right)^3 = \frac{2\pi\gamma m_p m_e M \kappa \varepsilon}{Le^2\hbar}$$

$$\Rightarrow r = R\sqrt[3]{\frac{2\pi\gamma m_p m_e M \kappa \varepsilon}{Le^2\hbar}} \approx 10^6 \text{ km}$$

Чернова

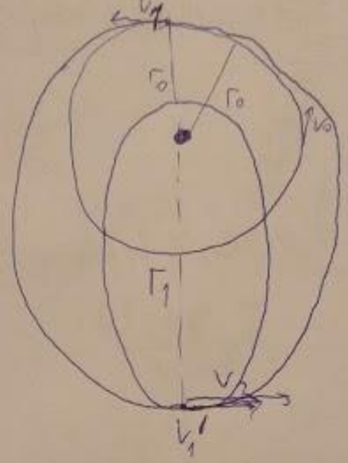
① $r_0 \approx 42200\,km$ $\left(\frac{r_0^3}{T_0^2} = \frac{\varkappa M}{4\pi^2}\right)$

План:

$3600 \cdot 24$
14400
$+7200$
86400

$42200 : 4 = 10550$

$42200 + 10550 = 52750$



$v_0 = \frac{2\pi r_0}{T} = \frac{2\pi \cdot 42200}{86400}$ ≈ 1, ≈ 2 (малко над)

$\Rightarrow v_0 \approx 3{,}1\,km/s$

$52750 \cdot 2 = 105500$

$105500 - 42200 = 63300$

$v_1 = v_0 + 10\%\, v_0 \approx 3{,}1 + 0{,}3 = 3{,}4\,km/s$

$$\left.\begin{array}{l} v_1 = \sqrt{\varkappa M\left(\frac{2}{r_0} - \frac{1}{a}\right)} \\ v_0 = \sqrt{\frac{\varkappa M}{r_0}} \end{array}\right\} \Rightarrow \frac{v_1}{v_0} = \sqrt{2 - \frac{r_0}{a}}$$

$$\frac{v_1^2}{v_0^2} = 2 - \frac{r_0}{a} \qquad \frac{r_0}{a} = 2 - \frac{v_1^2}{v_0^2}$$

$$\frac{v_1^2}{v_0^2} = \left(\frac{3{,}4}{3{,}1}\right)^2 = \left(1 + \frac{0{,}3}{3{,}1}\right)^2 \approx 1 + 2\cdot\underbrace{\frac{0{,}3}{3{,}1}}_{\approx 0{,}1} \approx 1{,}2$$

$$\Rightarrow \frac{r_0}{a} \approx 2 - 1{,}2 = 0{,}8 = \frac{8}{10} \qquad \frac{a}{r_0} \approx \frac{5}{4} = 1{,}25$$

$5 : 4 = 1{,}25$

$$a = 1{,}25\, r_0 \approx 52750\,km$$

$$a = \frac{r_0 + r_1}{2} \qquad r_1 = 2a - r_0 = 63300\,km$$

$$v_1' = \sqrt{\gamma M\left(\frac{2}{r_1} - \frac{1}{a}\right)} \qquad v_0 = \sqrt{\frac{\gamma M}{r_0}} \quad \Bigg\} \Rightarrow \frac{v_1'}{v_0} = \sqrt{\frac{2r_0}{r_1} - \frac{r_0}{a}} =$$

$$= \sqrt{\frac{2 \cdot 42200}{63300} - \frac{42200}{52750}} \approx \sqrt{2 \cdot \frac{2}{3} - \frac{4}{5}} =$$

$$= 2\sqrt{\frac{1}{3} - \frac{1}{5}} = 2\sqrt{\frac{5-3}{15}} = 2\sqrt{\frac{2}{15}} \approx 2\sqrt{0{,}13} \approx 2 \cdot 0{,}36 = 0{,}72$$

$2 : 15 = 0{,}13$
-0
20
-15
50
-45
5

$0{,}35 \cdot 0{,}35$
175
$+\ 105$
$0{,}1225$

$$v_1' = v_0 \cdot 0{,}72 = 3{,}1 \cdot 0{,}72 \approx 2{,}2\,km/s$$

$0{,}72 \cdot 3{,}1$
72
$+\ 216$
$2{,}232$

$$v_2 = v_1' - 10\% \cdot v_1' = 2{,}2 - 0{,}2 \approx 2\,km/s$$

$$v_2 = \sqrt{\gamma M\left(\frac{2}{r_1} - \frac{1}{a_f}\right)}$$

$$\frac{v_2}{v_0} = \sqrt{2 \cdot \frac{r_0}{r_1} - \frac{r_0}{a_f}} \qquad \frac{v_2^2}{v_0^2} = 2\frac{r_0}{r_1} - \frac{r_0}{a_f}$$

$$\frac{r_0}{a_f} = 2\frac{r_0}{r_1} - \frac{v_2^2}{v_0^2} \qquad \frac{v_2^2}{v_0^2} = \frac{2^2}{3{,}1^2} \approx \frac{4}{10} = 0{,}4$$

$$2\frac{r_0}{r_1} = 2 \cdot \frac{42200}{63300} \approx \frac{4}{3}$$

$4 : 3 = 1{,}333\ldots$
-3
10
-9
10

$$\frac{r_0}{a_f} \approx 1{,}3 - 0{,}4 = 0{,}9$$

$10 : 9 = 1{,}1$
-9
10
-9

$$\frac{a_f}{r_0} \approx 1{,}1 \qquad \frac{a_f^3}{r_0^3} = \frac{T_{fp}^2}{T_0^2}$$

$$\frac{T_{fp}}{T_0} = \left(\frac{a_f}{r_0}\right)^{\frac{3}{2}} = \left(\frac{1{,}1}{1}\right)^{\frac{3}{2}} = \left(1 + \frac{0{,}1}{1}\right)^{\frac{3}{2}} \approx 1 + \frac{3}{2} \cdot 0{,}1 = 1{,}15$$

Реално :

$u_1 = V_0 - 10\% V_0 =$

$= 3{,}1 - 0{,}31 \approx 2{,}8\,km/s$

$u_1 = \sqrt{\gamma M\left(\frac{2}{r_0} - \frac{1}{a}\right)}$

$\frac{u_1}{u_0} = \sqrt{2 - \frac{r_0}{a}}$

$$\frac{r_0}{a} = 2 - \left(\frac{u_1}{u_0}\right)^2 = 2 - \left(\frac{3{,}1 - 0{,}31}{3{,}1}\right)^2 = 2 - (1 - 0{,}1)^2 =$$

$$= 2 - (1 - 0{,}2) = 2 - 0{,}8 = 1{,}2$$

$$a = \frac{r_0}{1{,}2} \approx 0{,}8 r_0 = 33800\,km$$

$$\frac{42200 \cdot 8}{33760}$$

$$a = \frac{r_0 + r_1}{2} \qquad r_1 = 2a - r_0 = $$

$$= 6752 - 42200 = 25320\,km$$

$$\frac{33760 \cdot 2}{67520} \qquad \frac{\begin{array}{r}67520\\ -42200\end{array}}{25320}$$

$$u_1' = \sqrt{\gamma M\left(\frac{2}{r_1} - \frac{1}{a}\right)} \qquad \frac{u_1'}{V_0} = \sqrt{2\frac{r_0}{r_1} - \frac{r_0}{a}} =$$

$$= \sqrt{2 \cdot \underbrace{\frac{42200}{25320}}_{\approx 1{,}8} - \underbrace{\frac{42200}{33760}}_{\approx 1{,}33}} \approx \sqrt{2 \cdot 1{,}8 - 1{,}3} =$$

$$= \sqrt{3{,}6 - 1{,}3} = \sqrt{2{,}3} \approx 1{,}5$$

$1{,}5 \cdot 1{,}5 = 2{,}25$

$$u_1' \approx 1{,}5\,V_0 \approx 4{,}6\,km/s$$

$$u_2 = u_1' + 10\%\,u_1' = 4{,}6 + 0{,}46 = 5{,}06 \approx 5{,}1\,km/s$$

$$\frac{u_2}{V_0} = \sqrt{2\frac{r_0}{r_1} - \frac{r_0}{a_f}} \qquad \frac{r_0}{a_f} = 2\frac{r_0}{r_1} - \left(\frac{u_2}{V_0}\right)^2$$

$$\frac{2r_0}{r_1} = \frac{2 \cdot 42200}{25320} \approx 2 \cdot 1{,}8 \approx 3{,}6$$

$$\left(\frac{v_2}{v_0}\right)^2 = \left(\frac{5{,}1}{3{,}1}\right)^2 \approx \left(\frac{5}{3}\right)^2 = \frac{25}{9} \approx 2{,}7$$

$\cancel{5{,}0\,87}$ $\quad 25 : 9 = 2{,}7$

-18

$\;\;70$

-63

$\;\;\;7$

$$\Rightarrow \frac{r_0}{a_f} \approx 3{,}6 - 2{,}7 = 0{,}9 \quad \Rightarrow \frac{a_f}{r_0} \approx 1{,}1$$

$$\Rightarrow \frac{T_f}{T_0} \approx 1{,}15$$

$\Rightarrow$ Разликата е много малка, при тази точност на изчисленията е практически 0?

$0{,}01 \cdot 24h = 0{,}01 \cdot 86400s = 864s \approx 14min$

$864 : 60 = 14$

-60

$\;\;264$

-240

разликата е под 2.14min?

②

$1800:31 = 58$
-155
$\overline{250}$

$\frac{31.5}{155}$

$\frac{31.7}{217}$

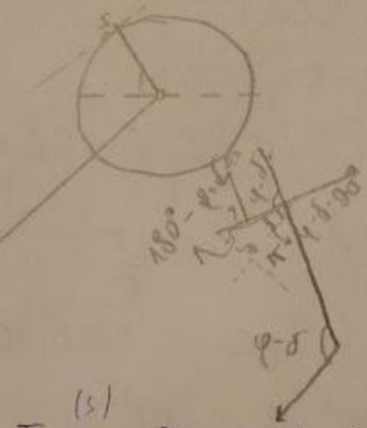
$\frac{57.2}{114}$

$\Delta\alpha = 5^{min} = \frac{1}{12} \cdot 15^\circ = 1{,}25^\circ$

$1 rad \approx 57^\circ$

$1{,}25 : 57 = 0{,}02$
-0
$\overline{12}$
-0
$\overline{125}$

$\frac{1{,}25}{57} \approx 0{,}02 \, rad$



αCMa

$\underline{\Delta\alpha \approx 0{,}02 \, rad}$

$\frac{\Delta\alpha^2}{2} = \frac{0{,}0004}{2} =$

$= 0{,}0002$

$1 - \frac{\Delta\alpha^2}{2} = 0{,}9998 \approx 1$

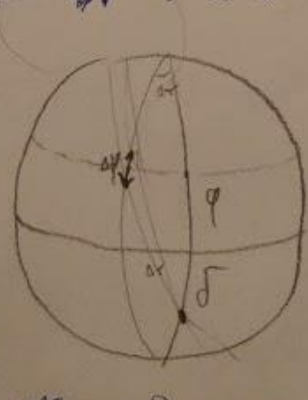
$21.12 \quad T_{3B} \overset{(s)}{=} T_{CA} + 6h$

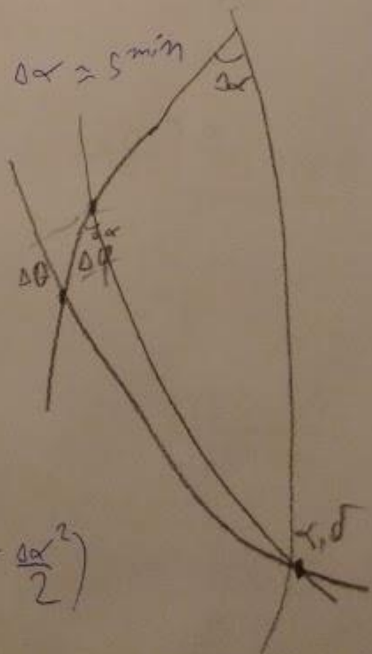
$1.01 - 21.12 = 10\text{-}11 d$

$10\text{-}11 d \cdot 4^m \approx 40^m$

$\Rightarrow \quad S \approx 6^h 40^m$

$\Delta\alpha \approx 5^{min}$





$\Delta\theta \approx \Delta\varphi \cos\Delta\alpha \approx$
$\approx \Delta\varphi\left(1 - \frac{\Delta\alpha^2}{2}\right)$

$\Delta\varphi = \frac{v \cdot t}{2\pi R} \cdot 360^\circ$

$\Delta S = \Delta\theta \cdot 2\pi R = \frac{vt}{\cancel{2\pi R}} \cdot \left(1 - \frac{\Delta\alpha^2}{2}\right) \cdot \cancel{2\pi R} = vt\left(1 - \frac{\Delta\alpha^2}{2}\right) = vt$

$\Delta S = 30 m$

$\Delta r = \Delta s \cdot \sin(\varphi - \delta) = \Delta s \cdot \sin 45^\circ = \frac{\Delta s}{\sqrt{2}} = \Delta s \cdot \frac{\sqrt{2}}{2} \approx 0{,}7 \Delta s$

$\dot{2}8$
$+17$
45°

$\Delta r \approx 21 m$

$M = m + 5 - 5 \lg r$

$m = M - 5 + 5 \lg r$

$\Delta m = 5 \lg \frac{r - \Delta r}{r} = 5 \lg \left(1 - \frac{\Delta r}{r}\right) \approx$

$\approx \cancel{8\left(\ldots\right)} \approx -5 \frac{\Delta r}{r}$

$r = 4 \,ly \approx 1{,}2 \,pc \approx 1{,}2 \cdot 2 \cdot 10^5 \,AU \approx 2{,}4 \cdot 10^5 \,AU =$

$\approx 2{,}4 \cdot 10^5 \cdot 150 \cdot 10^{12} \,m =$

$= 2{,}4 \cdot 1{,}5 \cdot 10^{19} \,m = 3{,}6 \cdot 10^{19} \,m$

$\Delta m = -5 \frac{\Delta r}{r} = -5 \cdot \frac{\cancel{21}\,7}{\cancel{36}\,12} \cdot 10^{-19} \approx -3{,}1 \cdot 10^{-19} \,m$

$35 : 12$

③

$\frac{r^3}{T^2} = M$

$r = \sqrt[3]{MT^2} = \sqrt[3]{2 \cdot 16} = \sqrt[3]{2 \cdot 2^4} =$

$= 2\sqrt[3]{4}\ AU \approx 2 \cdot 1{,}6 = 3{,}2\ AU$

$1{,}6 \cdot 1{,}6$
96
+16
2,56

~~2,56 · 2,56~~
~~1536~~
~~+1280~~
~~+512~~
~~6,5536~~

$20 \cdot 3600 =$

$2{,}56 \cdot 1{,}6$
+1536
256
4,096

$2{,}2 \cdot 10^4 \cdot 20 \cdot 3600 =$
$= 2{,}2\ 10^4 \cdot 2 \cdot 10^1 \cdot 3{,}6 \cdot 10^2 =$
$= 2{,}2 \cdot 2 \cdot 3{,}6 \cdot 10^8$

$3{,}6 \cdot 4{,}4$
144
+144
1584

$\frac{L}{L_\odot} \approx \left(\frac{M}{M_\odot}\right)^4 = 16$

$L_\odot \approx 3{,}8 \cdot 10^{26}\ W$

$I = \frac{L}{4\pi r^2}$ $P = IS$

$E = \eta I S t$

~~$E = \eta \frac{LSt}{4\pi r^2} = \frac{0{,}1 \cdot 16 \cdot 3{,}8 \cdot 10^{26} \cdot 100 \cdot 20 \cdot 3600}{4 \cdot 3{,}14 \cdot}$~~

$\frac{A}{A_\odot} = \frac{L}{L_\odot} \cdot \frac{1}{r^2} = 16 \cdot \frac{1}{3{,}2^2} \approx 1{,}6$

$A_\odot = 1367\ W/m^2$

$\Rightarrow A \approx 1{,}6 \cdot 1367 \approx 2200\ W/m^2$
$= 2{,}2 \cdot 10^3\ W/m^2$

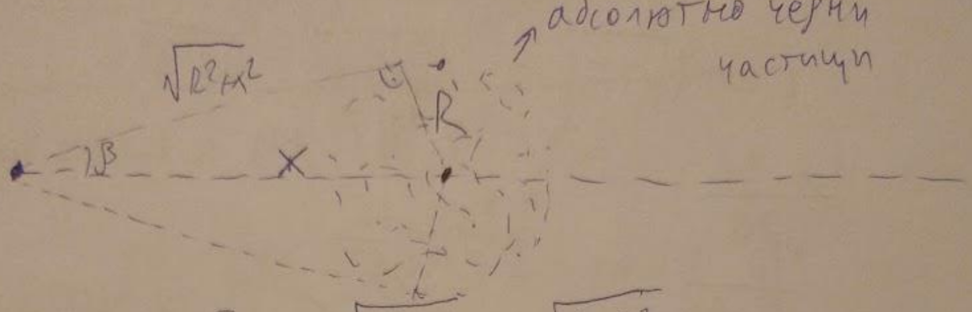
$1400 \cdot 1{,}6$
8400
+1400
22400

$P = A \cdot S = 2{,}2 \cdot 10^5\ W$

$P_{eff} = \eta P = 2{,}2 \cdot 10^4\ W$

$E = P_{eff} t = 2{,}2 \cdot 10^4 \cdot 20 \cdot 3600 \approx 1{,}6 \cdot 10^9\ J$

(4)



$$\cos\beta = \sqrt{\frac{R^2+x^2}{x}} = \sqrt{1+\frac{R^2}{x^2}}$$

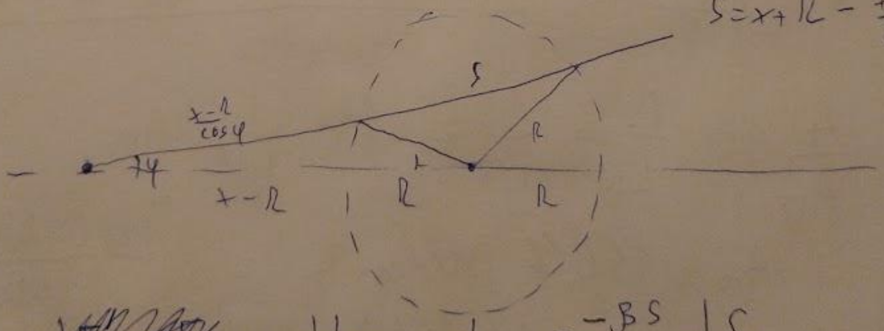
$$\Omega = 2\pi(1-\cos\beta) = 2\pi\left(1-\sqrt{1-\frac{R^2}{x^2}}\right)$$

$$\frac{L_n}{L_s} = \cancel{\frac{\Omega}{4\pi} = \frac{1}{2}}\left(1-\sqrt{1-\frac{R^2}{x^2}}\right)$$

$$\frac{L_n}{L_s} = k\cdot\frac{\Omega}{4\pi} = \frac{k}{2}\left(1-\sqrt{1-\frac{R^2}{x^2}}\right)$$

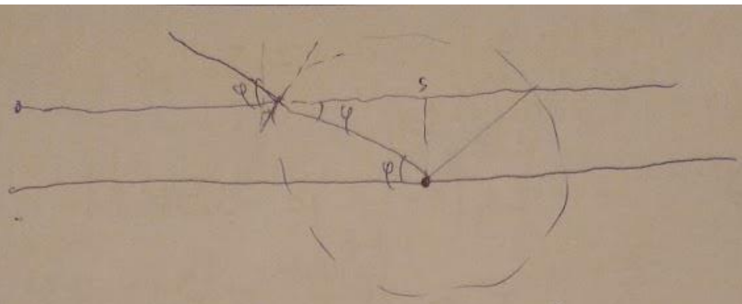
$$m = M - 5 + 5\lg r + A\cdot 2R$$

$$S = x + R - \frac{x-R}{\cos\varphi}$$



$$dL_n = L_s\cdot e^{-\beta S}\,dS$$

Нека $x \gg R$



$\cancel{s^2 = 2Rx}$ $\qquad s = 2R\cos\varphi$

$$dS = h \cdot R\,d\varphi \cdot \cos\varphi = hR\cos\varphi\,d\varphi$$

$$dE_n' = E_s e^{-\beta s} \cdot hR\cos\varphi\,d\varphi = E_s hR\,e^{-2\beta R\cos\varphi}\cos\varphi\,d\varphi$$

$12,0 - 7,7 = 4,3$

$$m_0 = M - 5 + 5\lg r = -2,7 - 5 + 5\lg 0,31$$

$\cancel{\lg 0,31 \approx 2 \cdot \frac{1-0,31}{1+0,31}}$

ℓ

$$\lg 310 = \lg 10 + \lg 31 = \approx 1 + 1,4 = 2,4$$

$\frac{2,4 \cdot 5}{12}$

$5,7 + 2,7 = 8,4$

$\cancel{-0,6 \cdot 5 = -3,0}$

$$m_0 = -2,7 - 5 + 12 = 4^m,3$$

$$E_n = \frac{L_n}{4\pi(r-x)^2} \qquad E_s = \frac{L_s - L_n}{4\pi r^2}$$

$$\frac{L_n}{\cancel{4\pi}(r-x)^2} = \frac{L_s - L_n}{\cancel{4\pi} r^2} \qquad \left(\frac{r-x}{r}\right)^2 = \frac{L_n}{L_s - L_n}$$

$$\underset{8,4}{\underset{22}{m - M}} = -2,5\lg\left(\frac{L_s - L_n}{\cancel{4\pi} r^2} \cdot \frac{\cancel{4\pi}(10pc)^2}{L_s}\right) = 5\lg\frac{r}{10_{=1,4}} - 2,5\lg\left(1 - \frac{L_n}{L_s}\right) = 7 - 2,5\lg\left(1 - \frac{L_n}{L_s}\right)$$

$$8{,}4 = 7 - 2{,}5\lg\left(1-\frac{L_n}{L_s}\right)$$

$$1{,}4 = -2{,}5\lg\left(1-\frac{L_n}{L_s}\right)$$

$$\frac{1{,}4\cdot 4}{5{,}6}$$

$$\lg\left(1-\frac{L_n}{L_s}\right) = -0{,}56$$

$$1-\frac{L_n}{L_s} \approx 10^{-0{,}56} \approx \frac{1}{3{,}3} \approx 0{,}35 \ ?$$

$$\frac{L_n}{L_s} = 0{,}65$$

$$\left(\frac{r-x}{r}\right)^2 = \cancel{0{,}6} \ \frac{\frac{L_n}{L_s}}{1-\frac{L_n}{L_s}} = \frac{0{,}65}{0{,}35} = 1{,}8$$

$$\begin{array}{r} 350 \\ -70 \\ \hline 280 \end{array}$$

$$65:35 = 1{,}8$$
$$\begin{array}{r} -35 \\ \hline 305 \\ -280 \end{array}$$

$$1-\frac{x}{r} \approx \left(1-\frac{0{,}2}{2}\right)^{\frac{1}{2}}$$

$$1-\frac{x}{r} \approx 1 - \frac{1}{2}\cdot\frac{0{,}2}{2}^{0{,}1} =$$
$$= 1 - 0{,}05$$

$$x \approx 0{,}05\,r =$$

$$\frac{310\cdot 0{,}05}{1{,}550} = 1{,}55\,pc$$

(5) $B(r) = B_0 \cdot \left(\frac{R}{r}\right)^3$

$$p(r) = \kappa B^2(r) = \kappa B_0^2 R^6 \cdot \frac{1}{r^6}$$

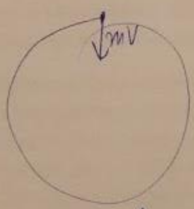
$$qvB_0 = \frac{mv^2}{r} \qquad \frac{v}{r} = \frac{2\pi c}{r} = 2\pi c \lambda = \frac{\mathcal{E}}{\hbar}$$

$$eB_0 = \frac{m_e \mathcal{E}}{\hbar} \qquad B_0 = \frac{m_e \mathcal{E}}{e\hbar}$$

$$NevB = Ne\sqrt{\frac{\gamma M}{r}}B =$$

$$= Ne\sqrt{\frac{\gamma M}{r}}\, B_0 \cdot \left(\frac{R}{r}\right)^3$$

$$L = \frac{dE_{gr}}{dt} = -\frac{\gamma M}{r}\frac{dm}{dt} = -\frac{\gamma M}{r}\frac{F}{v}$$



$$F = \frac{dp}{dt} = v\frac{dm}{dt} \qquad p = \frac{F}{4\pi r^2}$$

$$\frac{dm}{dt} = \sigma S v$$

$$F = v\frac{dm}{dt} = \frac{1}{\sigma S}\left(\frac{dm}{dt}\right)^2 = \frac{1}{\sigma S}\left(\frac{rL}{\gamma M}\right)^2$$

$$p = \frac{F}{S} = \frac{1}{\sigma}\left(\frac{rL}{\gamma M}\right)^2$$

$$F = NevB \qquad L = \frac{\gamma m M}{r^2}\frac{dr}{dt} = \frac{\gamma m M}{r^2}v$$

$$v = \frac{r^2 L}{\gamma m M} = \frac{r^2 L}{\gamma N m_p M}$$

$$F = \cancel{N}e \cdot \frac{r^2 L}{\gamma \cancel{N} m_p M} B = \frac{r^2 L e \cdot B}{\gamma m_p M}$$

$$p = \frac{F}{4\pi r^2} = \frac{Le}{4\pi\gamma m_p M} \cdot B$$

$$\frac{Le}{4\pi\gamma m_p M}\cancel{B} = \kappa B^{\cancel{2}}$$

$$\frac{Le}{4\pi\gamma m_p M} = \kappa \cdot \frac{m_e \varepsilon}{e t} \cdot \left(\frac{R}{r}\right)^3$$

$$\left(\frac{r}{R}\right)^3 = \frac{4\pi\gamma m_p m_e M \kappa \varepsilon}{L e^2 t}$$

$$r = R\sqrt[3]{\frac{\overset{2}{\cancel{4}}\pi\gamma m_p m_e M \kappa \varepsilon}{L e^2 t}}$$

$$10 \cdot \sqrt[3]{2 \cdot 3{,}14 \cdot 6{,}67 \cdot 10^{-11} \cdot \underline{\qquad} \cdot 2 \cdot 10^{31} \cdot 4 \cdot 10^{5} \cdot 10}$$