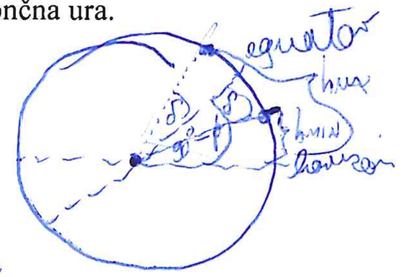
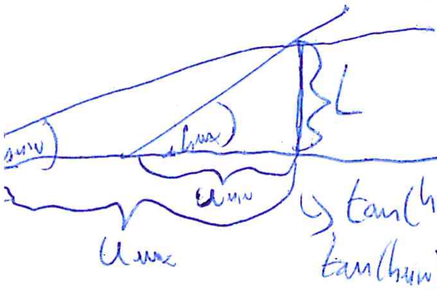
1. naloga

Gnomon (palica) horizontalne sončne ure je postavljen navpično. Med letom se dolžina opoldanske sence spremeni za dve dolžini gnomona. Izračunaj zemljepisno širino kraja, v katerem je ta sončna ura.

$\delta(t)$



$$\Rightarrow \boxed{\begin{aligned} h_{max} &= 90^\circ - \phi + \delta \\ h_{min} &= 90^\circ - \phi - \delta \end{aligned}}$$



$\Rightarrow \tan(h_{max}) = \frac{L}{u_{min}}$

$\tan(h_{min}) = \frac{L}{u_{max}}$

$$u_{max} - u_{min} = \frac{L}{\tan(h_{min})} - \frac{L}{\tan(h_{max})} = 2L$$

$$\Rightarrow \frac{1}{\tan(h_{min})} - \frac{1}{\tan(h_{max})} = 2$$

$$\Downarrow$$

$$\boxed{\frac{1}{\tan(90^\circ - \phi - \delta)} - \frac{1}{\tan(90^\circ - \phi + \delta)} = 2} \; ; \; \boxed{\delta \approx 23{,}5^\circ}$$

$$\frac{\cos(90^\circ - \phi - \delta)}{\sin(90^\circ - \phi - \delta)} - \frac{\cos(90^\circ - \phi + \delta)}{\sin(90^\circ - \phi + \delta)} = 2 \Rightarrow \frac{\sin(\phi + \delta)}{\cos(\phi + \delta)} - \frac{\sin(\phi - \delta)}{\cos(\phi - \delta)} = 2$$

$$\frac{\sin(\phi+\delta)\cos(\phi-\delta) - \sin(\phi-\delta)\cos(\phi+\delta)}{\cos(\phi+\delta)\cos(\phi-\delta)} = 2 \Rightarrow \frac{\sin(\phi+\delta-\phi+\delta)}{\cos(\phi+\delta)\cos(\phi-\delta)} = 2 \Rightarrow$$

$$\frac{\sin(2\delta)}{\cos(\phi+\delta)\cos(\phi-\delta)} = 2 \; ; \; \delta \approx 23{,}5^\circ \Rightarrow \cancel{\frac{\sin(47^\circ)}{\cos(\phi+47^\circ)\cos(\phi}} \quad \frac{\sin(47^\circ)}{\cos(\phi+23{,}5^\circ)\cos(\phi-23{,}5^\circ)} \approx 2$$

$$\boxed{\sin(47^\circ) \approx \frac{\sqrt{2}}{2}}$$

$$\frac{\sqrt{2}}{2} \cdot \frac{1}{\cos(\phi+23{,}5^\circ)\cos(\phi-23{,}5^\circ)} = 2 \Rightarrow \frac{\sqrt{2}}{4} = \cos(\phi+23{,}5^\circ)\cos(\phi-23{,}5^\circ)$$

$$\frac{\sqrt{2}}{4} = (\cos(\phi)\cos(23{,}5^\circ) - \sin(\phi)\sin(23{,}5^\circ))(\cos(\phi)\cos(23{,}5^\circ) + \sin(\phi)\sin(23{,}5^\circ))$$

$$\frac{\sqrt{2}}{4} = \cos(\phi)^2\cos(23{,}5)^2 - \sin(\phi)^2\sin(23{,}5^\circ)^2 \Rightarrow \frac{\sqrt{2}}{4} = \cos(\phi)^2\cos(23{,}5)^2 - \sin(23{,}5^\circ)^2 + \cos(\phi)^2 \cdot \sin(23{,}5^\circ)^2$$

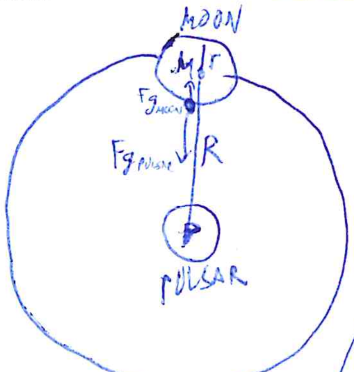
$$\frac{\sqrt{2}}{4} = \cos(\phi)^2(\sin(23{,}5)^2 + \cos(23{,}5)^2) - \sin(23{,}5)^2 \Rightarrow \cos(\phi)^2 = \frac{\sqrt{2}}{4} + \sin(23{,}5^\circ)^2$$

$$\sin(23{,}5^\circ) \approx \sqrt{\frac{1-\cos(45)}{2}} \approx \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} \approx \sqrt{\frac{1}{2} - \frac{\sqrt{2}}{4}} \Rightarrow \cos(\phi)^2 = \frac{\sqrt{2}}{4} + \frac{1}{2} - \frac{\sqrt{2}}{4}$$

$$\cos(\phi)^2 \approx \frac{1}{2} \Rightarrow \ldots \approx 1 \quad \Rightarrow \phi = 45^\circ$$

2. naloga

Leta 2003 so astronomi odkrili luno, ki kroži okoli pulzarja XTE J1807-294 (njegova masa je 1,4 mase Sonca). Obhodni čas lune okoli pulzarja je 0,03 dneva in ima maso 14,5 mase Jupitra. Kaj lahko ugotoviš o snovi, iz katere je ta luna? Svoje odgovore utemelji tudi z računi.



If Moon is stable:

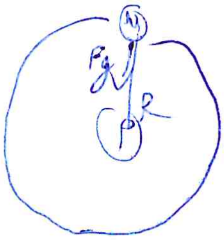
$$Fg_{MOON} = Fg_{PULSAR}$$

$$\frac{\cancel{G}\cancel{m_i} M_{MOON}}{r^2} = \frac{\cancel{G}\cancel{m_i} M_{PULSAR}}{(R-r)^2}$$

$$\boxed{\frac{M_{MOON}}{r^2} = \frac{M_{PULSAR}}{(R-r)^2}}$$

r → radius of Moon

R → distance from pulsar



$$F_g = F_c \Rightarrow \frac{G M_{PULSAR} \cancel{M_{MOON}}}{R^{\cancel{2}}} = \cancel{M_{MOON}} \cdot \frac{v^2}{\cancel{R}}$$

$$\frac{G M_{PULSAR}}{R} = \omega^2 \cdot R^2 \Rightarrow G M_{PULSAR} = \frac{4\pi^2}{T_0^2} \cdot R^3$$

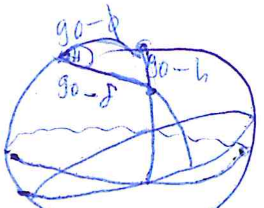
finding R → $\boxed{R^3 = \frac{G M_{PULSAR} T_0^2}{4\pi^2}} \approx \frac{6{,}67 \cdot 10^{-11} \cdot 1{,}4 M_\odot (0{,}03d)^2}{4\pi^2}$

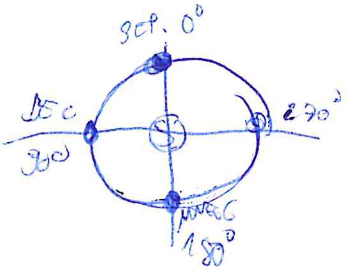
$$\frac{M_{MOON}}{r^2} = \frac{M_{PULSAR}}{(R-r)^2} \Rightarrow \cancel{r^2} = R \gg r \Rightarrow R - r \approx R$$

$$\frac{M_{MOON}}{r^2} \approx \frac{M_{PULSAR}}{R^2} \Rightarrow \boxed{r^2 = \frac{M_{MOON} R^2}{M_{PULSAR}}}$$ ← finding r

$$\rho_{MOON} = \frac{M_{MOON}}{V_{MOON}} = \frac{3 M_{MOON}}{4\pi r^3}$$

I can find DENSITY of material of moon



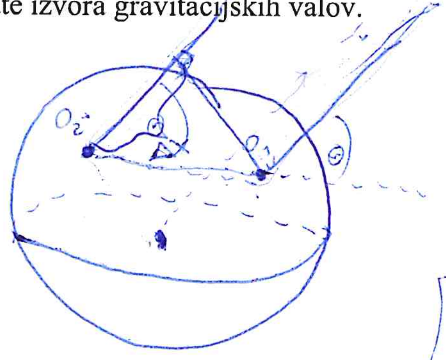


$\tan(30°) \approx \frac{1/2}{\sqrt{3}/2}$

3. naloga

Gravitacijski teleskopi LIGO v Livingstonu (30° 33' severne zemljepisne širine, 90° 47' zahodne zemljepisne dolžine) in Hanfordu (46° 27' severne zemljepisne širine, 119° 25' zahodne zemljepisne dolžine) in VIRGO (43° 38' severne zemljepisne širine, 10° 30' vzhodne zemljepisne dolžine) so 31. decembra ob 22.00 uri po univerzalnem času zaznali gravitacijske valove. Časovna razlika v prihodu valov med tremi teleskopi ni bila večja od $3x10^{-3}$ sekunde. V času pol ure za tem je ruski observatorij RAN (43° 40' severne zemljepisne širine, 41° 26' vzhodne zemljepisne dolžine) beležil zasij v vidni svetlobi, katerega izvor je bil izbruh sevanja gama, ki je bil tudi izvor gravitacijskih valov. Izračunaj približne ekvatorialne koordinate izvora gravitacijskih valov.

(δ, α)



$\Delta X \leq c\Delta t_{MAX}$

$\Delta X \leq 3\cdot10^8\cdot3\cdot10^{-3}$ m

$\boxed{\Delta X \leq 9\cdot10^5 \text{ m}}$

ob 22^{00} : viden ...

$\phi_1 \approx 30°$

$\cos(H) = -\tan(\phi)\tan(\delta)$

$\cos(H) = -\frac{1}{\sqrt{3}}\tan(\delta)$

$\cos(22^h + \lambda_1 + \alpha) = -\frac{1}{\sqrt{3}}\tan(\delta)$

$H = t_0 + \ldots$

$22^h + \lambda + \alpha$

30^{min} AFTER $\Rightarrow$ BEFORE GRAVITATIONAL WAVES SOURCE WAS BENEITH HORIZON IN RUSSIA

$\Downarrow$

$UT_{(\text{above horizon russia})} = 22^h 30^{min}$

$\cos(H_{RUSSIA}) = -\tan(\phi_{RUSSIA})\tan(\delta_{SOURCE})$

$H_{RUSSIA} = UT_{(\text{above horizon})} + S + \lambda_{RUSSIA} - \alpha_{SOURCE}$

$H_{RUSSIA} = 22^h 30^{min} + 12^h + 6^h - \alpha$

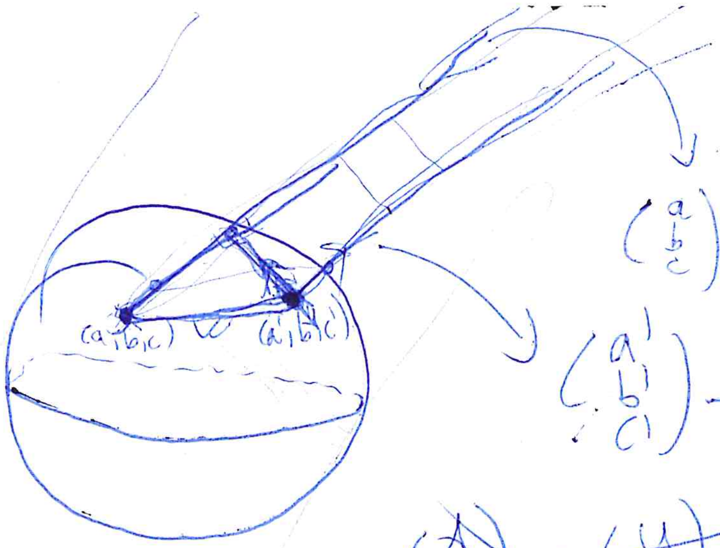
$H_{RUSSIA} = 16^h 30^{min} - \alpha$

$\Rightarrow \boxed{\cos(247° - \alpha) = -\cancel{\frac{\sqrt{2}}{2}} - \tan(\delta)}$ (I)

$S^{31.12} \approx S^{21.12} \approx 12^h$

$16{,}5 \cdot 15$

165
825
2475



$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} A \\ B \\ C \end{pmatrix} x$$

$$\begin{pmatrix} a' \\ b' \\ c' \end{pmatrix} + \begin{pmatrix} A \\ B \\ C \end{pmatrix} x$$

$$\cancel{\begin{pmatrix} A \\ B \\ C \end{pmatrix} \times \begin{pmatrix} u \\ g \\ h \end{pmatrix}} = \begin{pmatrix} A \\ B \\ C \end{pmatrix} \circ \begin{pmatrix} u \\ g \\ h \end{pmatrix} = 0$$

$$\begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix}$$

$$Au + Bg + Ch = 0$$

$$\sqrt{(R_x - a)^2 + (R_y - b)^2 + (R_z - c)^2} - \sqrt{(R_x - a')^2 + (R_y - b')^2 + (R_z - c')^2} = d$$

$$\cancel{d^2 = (R_x - a)^2 + (R_y - b)^2 + (R_z - c)^2 - (R_x - a')^2 - (R_y - b')^2}$$

$$\cancel{d^2 = (R_x - a)^2 + (R_y - b)^2 + (R_z - c)^2 + (R_x - a')^2 + (R_y - b')^2} + (R_z - c')^2 -$$

$$\cancel{\sqrt{2}}$$

$$Au + Bg + Ch = 0$$

$$\begin{pmatrix} u \\ g \\ h \end{pmatrix} x + \begin{pmatrix} a' \\ b' \\ c' \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} A \\ B \\ C \end{pmatrix} x$$

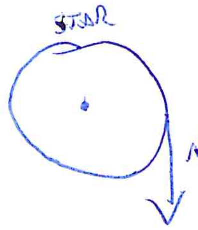
$$\begin{pmatrix} a - a' \\ b - b' \\ c - c' \end{pmatrix} = x \begin{pmatrix} u - A \\ g - B \\ h - C \end{pmatrix}$$

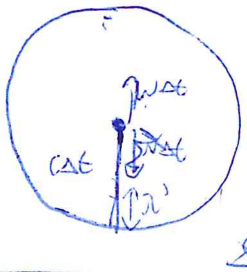
KAKO HITRO SE VRTI (I)

NA VEČ STRANEH

4. naloga

Astronomi so v spektru neke zvezde opazovali absorpcijsko spektralno črto titanovega oksida, ki ima laboratorijsko valovno dolžino 5170,7 Å (1 Å = 10^{-10} m). Valovna dolžina te spektralne črte v središču ploskvice zvezde je bila 5174,1 Å, na robu ploskvice na ekvatorju zvezde pa 5174,2 Å. Gostota zvezde je 0,7 g/cm^3. Oceni najmanjši možni izsev te zvezde.





$\lambda' = \frac{c}{f_0} + \frac{v}{f_0}$

$\lambda \cdot f = c$

$\frac{1}{f} = \frac{\lambda}{c}$

$\lambda' = \frac{1}{f_0}(c+v)$

$\lambda' = \frac{\lambda}{c}(c+v)$

$\lambda' = \lambda\left(1+\frac{v}{c}\right)$

~~$\lambda < 5$~~

It seems like STAR ITSELF is moving away from us

$\lambda = 5174{,}1 Å$

$\lambda' = 5174{,}2 Å \Rightarrow \cancel{0{,}2 Å}$

$\cancel{0{,}1 Å = 1 + \frac{v}{300\,000\,000 \frac{m}{s}}}$

$\cancel{0{,}1 \cdot 10^{-10} m = 1 + \frac{v}{3 \cdot 10^8 \frac{m}{s}}}$

$\left(\frac{5174{,}2}{5174{,}1} - 1\right)c = v_{ROT}$

$(0{,}99999 - 1)c = v_{ROT} \Rightarrow 10^{-5} \cdot 3 \cdot 10^8 \frac{m}{s} = v_{ROT} \Rightarrow v_{ROT} \approx 3 \cdot 10^5 \frac{m}{s}$

$-0{,}00001\,c = v_{ROT}$

(velocity of rotation)

$F_g = m_i \cdot \frac{v^2}{r_{STAR}} \Rightarrow \frac{G m_i \cdot M_{STAR}}{r_{STAR}^2} = m_i \frac{v^2}{r_{STAR}}$

$v^2 = \frac{G M_{STAR}}{r_{STAR}} \Rightarrow \boxed{\left(\frac{M_{STAR}}{r_{STAR}}\right)_{MIN} = \frac{v^2}{G}}$

$\rho_{STAR} = \frac{M_{STAR}}{V} = \frac{M_{STAR} \cdot 3}{4\pi r^3} \Rightarrow r = \sqrt[3]{\frac{3 M_{STAR}}{4\pi \rho_{STAR}}}$

$\frac{M_{STAR} \cdot \sqrt[3]{4\pi \rho_{STAR}}}{\sqrt[3]{3 \cdot M_{STAR}}} = \frac{v^2}{G} \Rightarrow M_{STAR}^{\frac{2}{3}} \approx \frac{v^2}{G} \cdot \sqrt[3]{\frac{3}{4\pi \rho_{STAR}}}$

$M_{STAR} =$

$$M_{STAR}^{\frac{2}{3}} \approx \frac{v^2}{G} \sqrt[3]{\frac{3}{4\pi \rho_{STAR}}}$$

$\underbrace{3\cdot3}_{9}\cdot\underbrace{3\cdot3}_{9}\cdot\underbrace{3\cdot3}_{9}$

$\underline{81\cdot9}$

$\underline{729} \approx 730$

$\underline{7{,}3\cdot10^2\cdot10^{30}}$

$$M_{STAR}^2 \approx \frac{v^6}{G^3}\cdot\frac{3}{4\pi\rho_{STAR}} \Rightarrow M_{STAR}^2 \approx \frac{7{,}3\cdot10^{32}}{3\cdot10^{-31}} : 1{,}4\cdot10^{-01}$$

$$M_{STAR}^2 \approx 3{,}5\cdot10^{62} kg \Rightarrow M_{STAR} \approx 1{,}8\cdot10^{31} kg$$

SS

$\underline{2\cdot10^{31} kg}$

$$\boxed{\text{It is true, that: } L \propto M^{3,5}}$$

~~we ko~~ we know, that $M_\odot \approx 2\cdot10^{30} kg$

$$\Downarrow$$

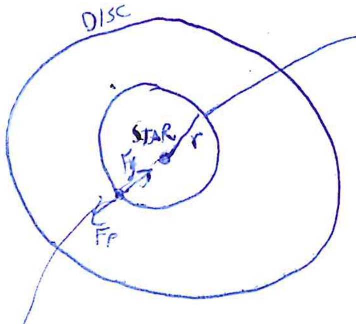
$$\left(\frac{2\cdot10^{31}}{2\cdot10^{30}}\right)^{\frac{3}{5}} \approx \frac{L_{STAR}}{L_\odot} \Rightarrow \left(\frac{10^{31}}{10^{30}}\right)^{\frac{3}{5}} \approx \frac{L_S}{L_\odot}$$

$$\Downarrow$$

$$10^{\frac{3}{5}} \approx \frac{L_S}{L_\odot} \Rightarrow \boxed{L_{S_{MIN}} \approx 10^{\frac{3}{5}} L_\odot}$$

5. naloga

Protoplanetarni disk je zelo tanek disk snovi, ki kroži okoli mlade zvezde. Predpostavi, da je disk v termodinamičnem in hidrostatičnem ravnovesju in najdi odvisnost gostote snovi nad ravnino diska od oddaljenosti r od zvezde. Masa zvezde M, temperatura diska T in molska masa snov μ so znane količine.



$$p \cdot V = n \cdot R \cdot T$$

$$p \cdot 4\pi r^2 dr = \frac{dm_{(r)}}{\mu} RT$$

$$p \cdot \cancel{4\pi r^2 dr} = \frac{\rho_{(r)} \cancel{4\pi r^2 dr}}{\mu} RT$$

$$p = \frac{\rho_{(r)}}{\mu} RT \Rightarrow dp = \frac{1}{\mu} RT \, d\rho_{(r)}$$

$$F_g = F_p$$

$$F_g = \frac{G\, dm \cdot M_r}{r^2} = \frac{G \cdot 4\pi \cancel{r^2} dr \cdot \rho_{nar} \cdot M_r}{\cancel{r^2}} = G \cdot 4\pi \rho_{nar} M_r \, dr =$$

$$G 4\pi \rho_{nar} \, dr (M_{STAR} + M_{DISC}) \leftarrow F_g$$

$$F_p = A \cdot dp = 4\pi r^2 \cdot dp = 4\pi r^2 \cdot \frac{1}{\mu} RT \, d\rho_{(r)}$$

$$F_g = F_p$$

$$\cancel{4\pi} G \rho_{nar} \, dr (M_{STAR} + M_{DISC}) = \cancel{4\pi} r^2 \frac{1}{\mu} RT \, d\rho_{(r)}$$

$$\boxed{M_{STAR} \gg M_{DISC} \Downarrow M_{STAR} + M_{DISC} \approx M_{STAR}}$$

$$\rho_{nar} M_{STAR} \, dr = \frac{1}{\mu} r^2 RT \, d\rho_{(r)}$$

$$\frac{1}{\rho_{nar}} d\rho_{nar} = \frac{G M_{STAR} \cdot \mu}{RT} \int_0 \frac{1}{r^2} dr$$

$$\left[\ln(\rho)\right]_{\rho_0}^{\rho_k} = \frac{G M_{STAR} \mu}{RT} \left[-\frac{1}{r}\right]_{R_{STAR}}^{r}$$

$$\ln(\rho_k) - \ln(\rho_0) = \frac{G M_{STAR} \mu}{RT} \left(\frac{1}{r_k} - \frac{1}{r_{STAR}}\right)$$

$$\ln\left(\frac{\rho_k}{\rho_0}\right) = \frac{G M_{STAR} \mu}{RT} \left(\frac{1}{r_k} - \frac{1}{R_S}\right)$$

$$\Rightarrow \boxed{\rho_k = \rho_0 \cdot e^{\frac{G M_{STAR} \mu}{RT}\left(\frac{1}{r_k} - \frac{1}{R_{STAR}}\right)}}$$