

$$\begin{array}{r} 43.57000 \\ \hline 7,4 \end{array}$$

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$$\begin{array}{r} 57000 \cdot 43 \\ \hline 228000 \\ 171000 \\ \hline 2451000 \\ \hline 74 \end{array}$$

$$2451000 : 74 = 702$$

Short Solutions:

Diameter of a moon: 600km

Period of moon: 5h

Period of conjunction: 0,22d

What would happen if Titan had the same orbit as moon? See big yellow sign.



$$\frac{r(r_y - r_x)}{x}$$

$$\begin{array}{r} 0,3 \cdot 7,4 \\ \hline 2,22 \\ \hline 0,5 \end{array}$$

$$\frac{77 \cdot 26 \cdot 28 \cdot 9}{58 \cdot 26} = \frac{3}{2}$$

for

$$\begin{array}{r} 57000 \cdot 7,4 \\ \hline 7,5 \end{array}$$

$$\begin{array}{r} 57000 \cdot 7,5 \\ \hline 7,4 \end{array}$$

28

270000

$$\frac{256 \cdot 10^{15}}{7,7 \cdot 10^{24}}$$

$$\begin{array}{r} 0,21 \cdot 24 \\ \hline 48,4 \\ \hline 5,04 \end{array}$$

$$\begin{array}{r} 2,56 \\ \hline 1,7 \cdot 10 \end{array}$$

2,56

25

$$26:17 = 2$$

0,7

57

$$\begin{array}{r} 1,5 \cdot 60000 \\ \hline 300000 \end{array}$$

$$300000 : 24$$

$$450000 : 17$$

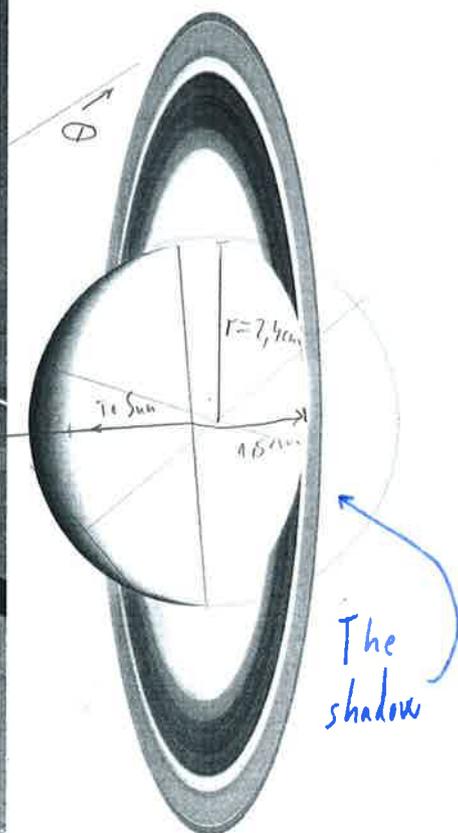
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Na prvi fotografiji je vidna Saturnova luna, ki se giblje v zunanem območju kolobarjev, na drugi pa je Saturn v negativu. Znano je, da je v času opazovanja luna bila v ravnini, ki je pravokotna na kolobarje in hkrati na zveznici med središčema Sonca in Saturna. Kot med ravnino kolobarjev in smerjo proti Soncu je 1 stopinja. Polmer Saturna je 9-krat večji od polmera Zemlje.

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Oceni premer lune in njegov obhodni čas okoli Saturna. Vsake koliko časa je ta luna v konjunkciji s Saturnovo luno Titan? Titan se okoli Saturna giblje po krožnici s polmerom 1,2 milijona kilometrov, njegov obhodni čas pa je 16 dni. Opiši, kaj bi se zgodilo, če bi se Titan nahajal na isti orbiti kot opazovana luna.

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The remark on symbols:

☉ - Sun

⊕ - Earth

♃ - Saturn's moon

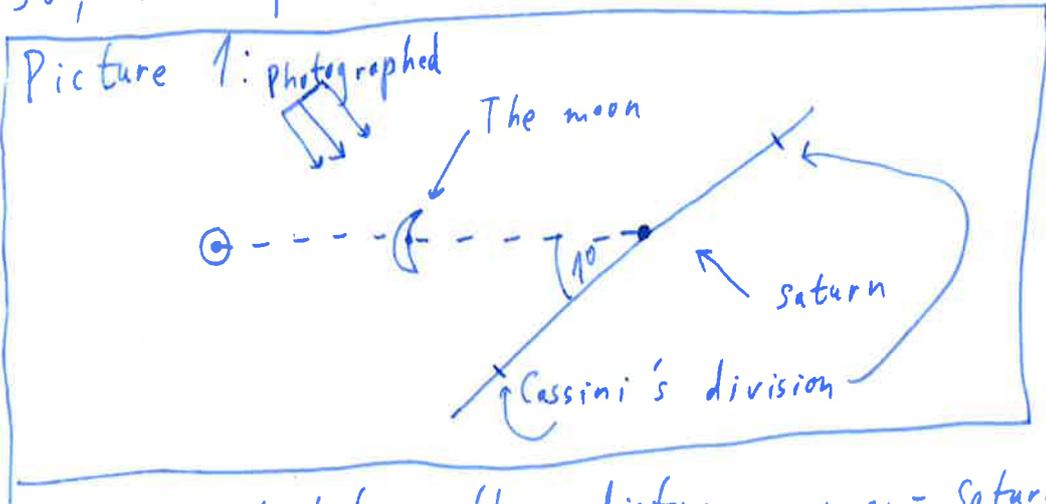
♃ - Titanus

♄ - Saturn

Note:
These are not the official sign (just because of simplicity)

FOR THE SOLUTION FOLLOW THE LINE

Let's say, that the moon is higher than the rings (it doesn't matter, it's equivalent)
 So, the picture is as follows:



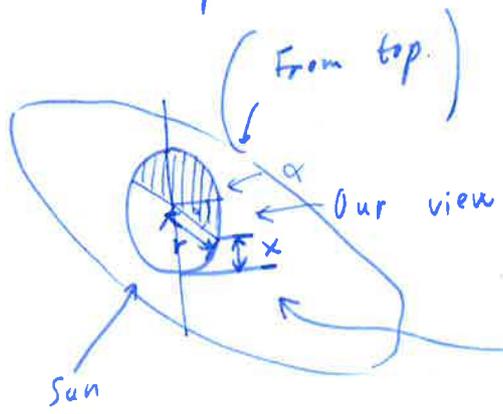
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Let's calculate the distance moon - Saturn's center. (=a)

$r_{\oplus} = 6380 \text{ km}$

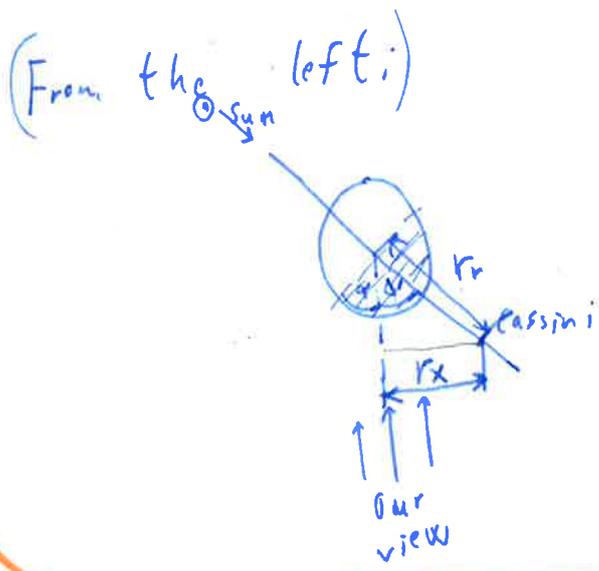
$r_{\oplus} = 6380 \text{ km} \cdot 9 = 57420 \text{ km} \approx \underline{57000 \text{ km}}$

From the picture, we get:



$\alpha = \arcsin\left(\frac{r-x}{r}\right)$

~~inclination~~ $\text{inclination} = \cancel{1^\circ} \approx \underline{0^\circ}$



IN RADIANS

$$r_r = \frac{r_x}{\sin \alpha} = \frac{r_x}{\sin \arcsin \frac{r-x}{r}} =$$

$$= \frac{r_x \cdot r}{x} = \frac{1,1 \text{ cm} \cdot 2,9 \text{ cm}}{0,5 \text{ cm}} =$$

$$= 4,28 \text{ cm}$$

GO TO
 1

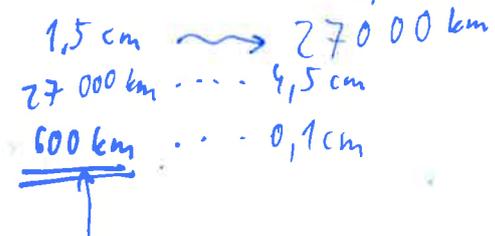
2

From the previous equations we get:



$$x = \frac{r_y \cdot r}{x} - \frac{r_x \cdot r}{x} = \frac{r(r_y - r_x)}{x}$$

$$= \frac{2,4 \text{ cm} (1,4 \text{ cm} - 1,1 \text{ cm})}{0,5 \text{ cm}} = 1,5 \text{ cm}$$



Result: 600 km is size of moon.

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1

This corresponds to:



BUT...

The instructions weren't true at all. They say this:



But I can see the shadow on the rings. So the moon isn't entirely on the line.

Let's calculate k for Titan:

$$k = \frac{a^3}{P^2} = \frac{(1,2 \cdot 10^8 \text{ km})^3}{(16 \text{ d})^2} = \frac{1,7 \cdot 10^{18} \text{ km}^3}{256 \text{ d}^2}$$

Now the period for the unknown moon:

$$P = \sqrt{\frac{a^3}{k}} = \sqrt{\frac{(10,2 \cdot 10^4 \text{ km})^3 \cdot 256 \text{ d}^2}{1,7 \cdot 10^{18} \text{ km}^3}} = \underline{\underline{0,21 \text{ d} = 5 \text{ h}}}$$

$$\left| \frac{1}{P_1} - \frac{1}{P_2} \right| = \frac{1}{P_s}$$

$$\left| \frac{1}{16 \text{ d}} - \frac{1}{0,21 \text{ d}} \right| = \frac{1}{P_s} = \frac{1}{5 - 0,05} \Rightarrow \underline{\underline{d = 0,22 \text{ d}}}$$

VERY SMALL

If these two moons were on the same orbit, there would nothing happen. (Like by the Trojans and the Greeks in Jupiter orbit.) But if they were very close, they will collide and at some time they will form a new moon.

GO TO 2

$$\frac{729 \cdot 20^{12} \cdot 256}{1,7 \cdot 10^{28}}$$

$$\frac{9 \cdot 9}{81 \cdot 9} = \frac{81}{729}$$

$$\frac{7,2 \cdot 7,2}{1,4 \cdot 1,2} = \frac{78}{1,68}$$

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1,7 km

$$\frac{1,1 \cdot 2,4}{0,5}$$

$$\frac{7,29 \cdot 2,56 \cdot 10^4}{1,7 \cdot 10^{12}}$$

$$\left(\frac{4,2 \cdot 10^6 \text{ km}^2}{(16^2 d)^2} \right)$$

$$\frac{16 \cdot 16}{256}$$

$$\frac{1,7 \cdot 10^{18} \text{ km}^3}{256 d^2}$$

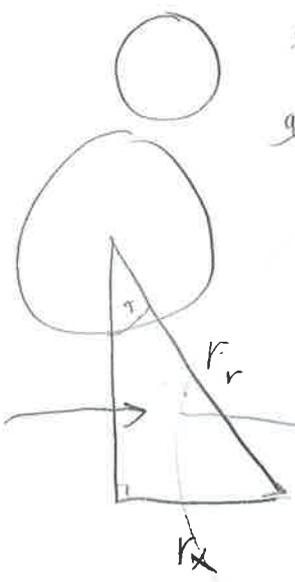
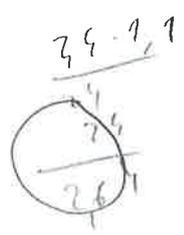
$$\frac{7,3 \cdot 2,6}{1,7 \cdot 10^2} d =$$

$$\frac{7,64 \cdot 2}{4,28}$$

$$\frac{0,192 \cdot 24}{224} = \frac{4,48}{21688}$$

$$\frac{7,3 \cdot 2,6}{146} = \frac{438}{18,98}$$

$$1,1 \cdot 10^{-2}$$



$\sin \alpha = \frac{r_x}{r}$
 $\frac{1,5 \cdot 2,4}{2,74} = 0,5$

$$19 : 1,7 = 0,112$$

$$150 : 77 = 1,948$$

$$\alpha = \arcsin\left(\frac{r-x}{r}\right)$$

$$\frac{r_x}{r} = \frac{x}{r}$$

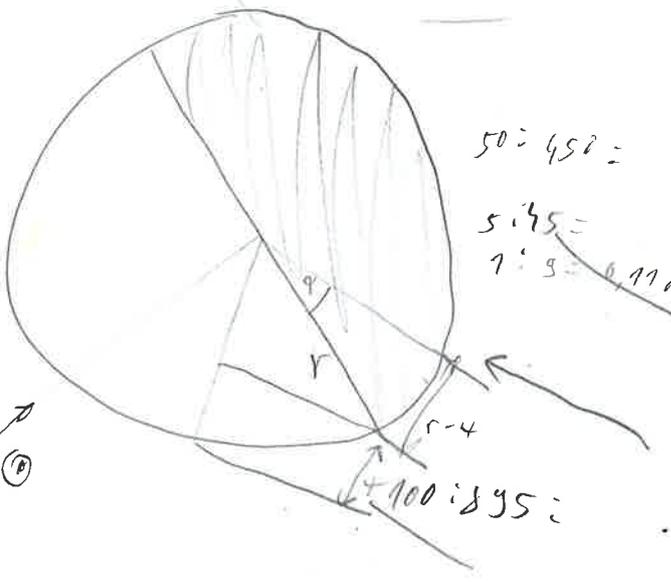
$$r_r = \frac{r_x}{\sin \alpha}$$

$$\frac{r_x \cdot r}{x}$$

$$\frac{1,5 \cdot 2,4}{3}$$



$$\frac{r \cdot x}{(r-x)} = \operatorname{arsh}\left(\frac{x}{r}\right)$$



$$50 : 450 =$$

$$5 : 45 = 1 : 9 = 0,111 d$$

$$1 : 1000 = 1 : 992 =$$

$$500 : 56 =$$

$$750 : 28$$

$$125 : 13 = 9$$

$$\frac{1,5 \cdot 2,4}{1,16}$$

$$\frac{r_x \cdot r}{x}$$

$$\frac{1,5 \cdot 2,4}{1,16} = 0,04$$

$$\frac{1,5 \cdot 2,4 \cdot 2}{1,16}$$

$$9 - 1,5$$

$$\frac{1,5 \cdot 2,4}{2}$$

$$8,95$$

$$90 - \alpha$$

$$r_r = \sin \left(90 - \arcsin \left(\frac{r-x}{r} \right) \right) \cdot r_x$$

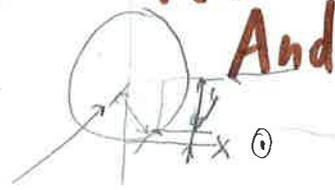
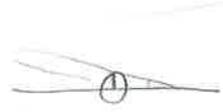
$$\sin \left(90 - \arcsin \left(\frac{0,5 \text{ cm} - 1,5 \text{ cm}}{0,5 \text{ cm}} \right) \right)$$

$$\sin \left(90 - \arcsin \left(\frac{2,4 \text{ cm} - 0,5 \text{ cm}}{2,4 \text{ cm}} \right) \right) \cdot 1,5 \text{ cm}$$

$$\sin \alpha = \frac{r_x}{r}$$

$$\sin \alpha' = \frac{r_r}{r_x}$$

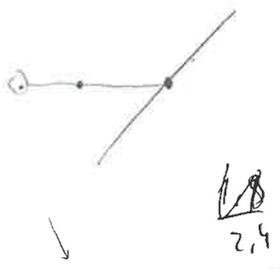
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4

r

$$r_r = \sin \alpha \cdot r_x$$



r-x

$$\frac{1,5}{2,4}$$

$$\frac{110,9}{2,4} = 45,7$$

a

$$\frac{1,5 \text{ cm} \cdot 6}{5}$$

$$\frac{6380,5}{57420}$$

$$\frac{28000}{57000} \cdot 3,5$$

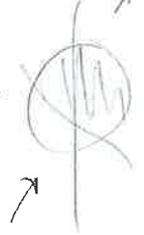
$$\frac{1,5}{2,4} \cdot 1,7$$

$$\frac{13}{24}$$

$$\frac{1,5 \cdot 5}{4}$$

$$1,5$$

110



60000

$$\frac{28000 \cdot 3,5}{17}$$

$$\frac{28000 \cdot 3,5}{84000} + \frac{84000}{940000} = \frac{380000}{940000}$$

50000



$$\frac{19 \cdot 480}{24 \cdot 11} = \frac{1,5 \cdot 0,8}{120}$$

$$\frac{2160 - 19}{24}$$

$$\frac{24 \cdot 90}{2760}$$

$$\frac{11}{2} = \frac{5}{8}$$

$$19:24$$

$$1,6 - 0,8 = 0,8$$

$$\sin(89) = 1,1,5 \text{ cm}$$

$$\frac{36 \cdot 6}{276}$$

$$5:6 = 0,8$$

$$625:125 = 5$$

115

89

0,8

20

$$\frac{5 \cdot 280}{8 \cdot 11} = 30$$

$$x = \frac{x^3}{3}$$

$$\tan 1^\circ = 1$$

$$\frac{150}{3} = 50$$

$$\frac{24 \cdot 3}{848}$$

$$\frac{125}{5}$$

$$\frac{925}{650}$$

625

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