XXVIII Санкт-Петербургска олимпиада
по Астрономия
Теоретичен тур
2. II. 2020

Задача 1. Щом едва се вижда:

$m_{max} \approx 6^m$

$\frac{L_{max}}{L_{min}} = 10^{0.4(m_{min} - m_{max})} \approx 10^4$

$L = 4\pi R^2 \sigma T^4$, $T = const$

$\Rightarrow L \sim R^2$

$\frac{R_{max}^2}{R_{min}^2} = 10^4$ $\frac{R_{max}}{R_{min}} = 100$

За половин пулсация $\left(t = \frac{P}{2} \approx 200^d\right)$

радиусът се промени с $\Delta R = R_{max} - R_{min}$

$\Delta R = 99 R_{min} \approx 100 R_{min} \approx R_{max}$

Знаем, че $5 \cdot 10^2 R_\odot$ е радиусът или в максимум, или в минимум.

Ако $R_{min} = 5 \cdot 10^2 R_\odot$

$v = \frac{\Delta R}{t} \approx 2,4 \cdot 10^3 km/s$ $(R_\odot \approx 7 \cdot 10^5 km)$

Този случай обаче не е реалистичен, тъй като означава $R_{max} = 5 \cdot 10^4 R_\odot = 230 AU$, което е твърде голям размер за звезда

Ако $R_{max} = 5 \cdot 10^2 R_\odot$

$v = \frac{\Delta R}{t} \approx \frac{R_{max}}{t} \approx 24 km/s$

Задача 2: $M_{O_2} = 2\cdot 16 g/mol = 0,032 kg/mol$

$N_A \approx 6,2\cdot 10^{23} mol^{-1}$

Масата на атмосферата е:

$$m = nM_{O_2} = \frac{N}{N_A} M_{O_2} \approx 1,3\cdot 10^{4} kg$$

Това е много малка маса за атмосфера. Можем да предположим, че по-голямата част от нея се намира до височина

$h \sim 10^0 - 10^1 km \ll R$ ($R = 764 km$).

$$g \approx \frac{\gamma M}{R^2}, \quad M = \rho\cdot\frac{4}{3}\pi R^3$$

$$S = 4\pi R^2$$

$$p = \frac{mg}{S} = \frac{m}{4\pi R^2}\cdot\frac{\gamma}{R^2}\cdot\rho\cdot\frac{4}{3}\pi R^3$$

$$p = \frac{\rho\gamma m}{3R} \approx 5\cdot 10^{-10} Pa$$

Задача 3. Моля, вижте черновата.

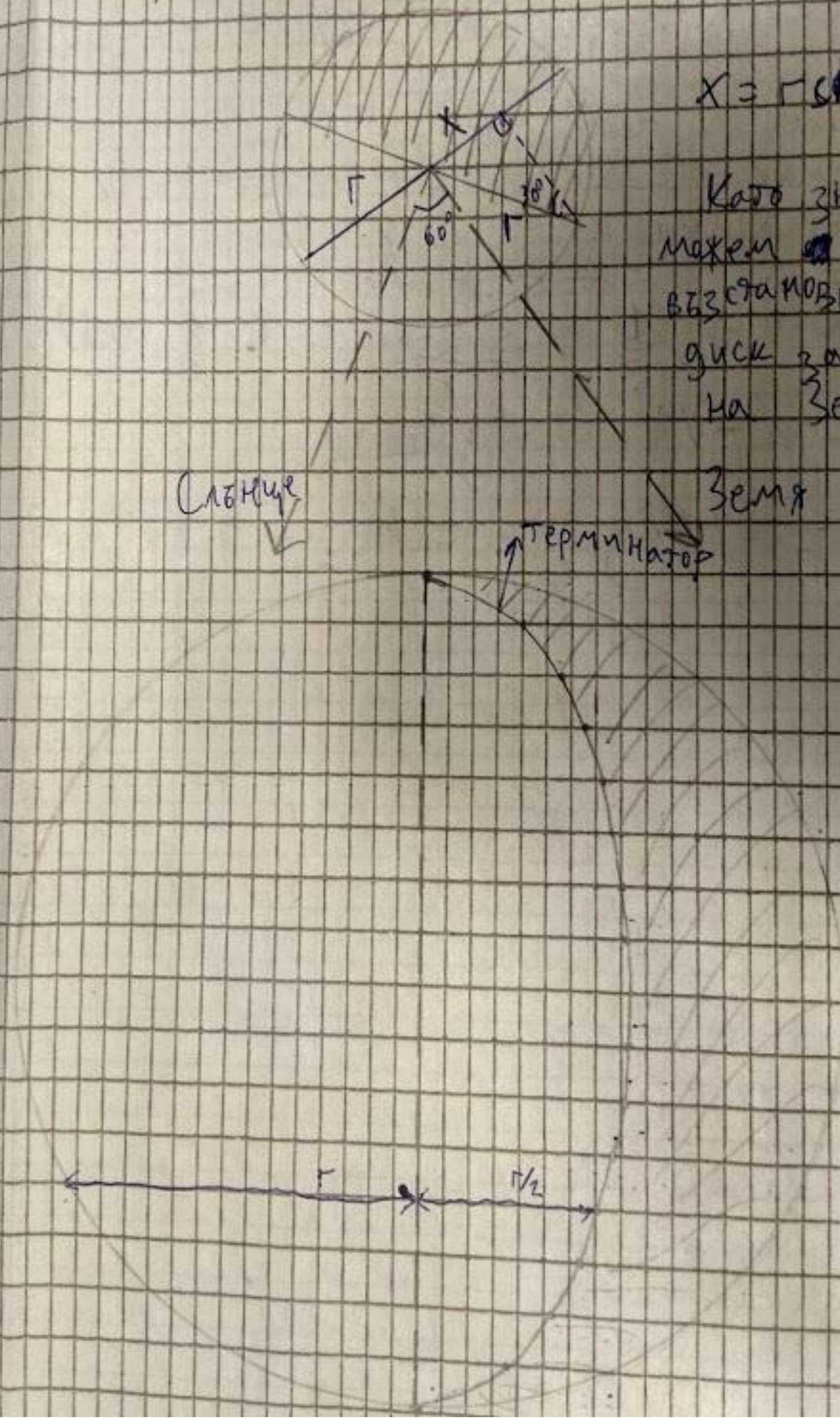
5.01 11ч − 2.01 4ч = 79ч

$\frac{79ч}{20ц} = 3,9$ ч/ц е промяната в част на преминаване през перихелий

Задача 4. Нека начертаем даден „елиптичен паралел", погледнат отгоре:

$$x = r\sin 30^\circ = \frac{r}{2}$$

Като знаем това, можем графично да възстановим видимия диск за наблюдател на Земята



В неосветената част на тъмно влизат 77 правоъгълника, 57 са на границата, т.е. ефективно се побират около $\frac{(77+57)+57}{2} \approx 290$

Всеки е с площ $8.4 = 32\ mm^2$

$S_{тъмно} = \frac{290}{2} \cdot 32 \approx$ ~~$9280\ mm^2$~~ 4640

Радиуса на кръга е $\frac{R}{2} \approx 66\ mm$,

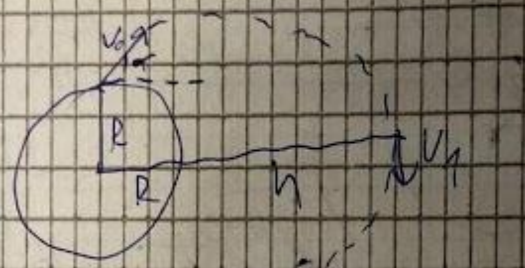
$S = \pi R^2 \approx$ ~~$6825\ mm^2$~~ $13505\ mm^2$

$\frac{S_{тъмно}}{S} \approx \frac{1}{3}$, $\frac{S_{светло}}{S} = 1 - \frac{1}{3} = \frac{2}{3} \approx 0,67$

$\Delta m = m - M = -2,5 \lg \frac{S_{светло}}{S} =$

$= -2,5 \lg \frac{2}{3} = 2,5 \lg \frac{3}{2} \approx 0,25^m$

Задача 5. Оказва се, че е най-ефективно модулът да излети право нагоре ~~(моля, вижте черновата)~~. В случая въртенето на Луната около оста ѝ оказва много слабо влияние.



$$m v_0 \cos\alpha \, R = m v_1 (R+h)$$

$$\frac{m v_0^2}{2} - \frac{\gamma m M}{R} = \frac{m v_1^2}{2} - \frac{\gamma m M}{R+h}$$

След преобразувания:

$$v_0^2 \approx \frac{2\gamma M h}{R^2\left(1-\cos^2\alpha \frac{R^2}{(R+h)^2}\right)}$$

Минималната стойност (при $\alpha = \frac{\pi}{2} = 90^\circ$)

е $v_0 = \sqrt{\frac{2\gamma M h}{R^2}} \approx 1,5 km/s$

Орбитата ще е линия, аналогична на елипса с $e = 1$ и $a = \frac{R+h}{2}$

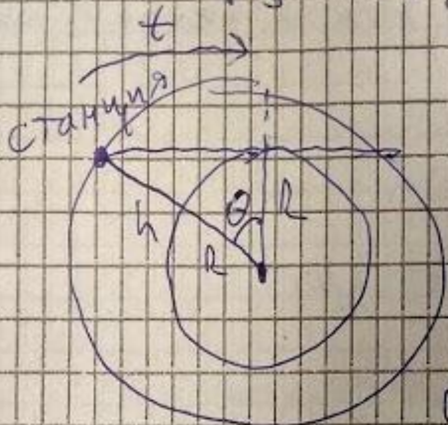
$$\left(\frac{a^3}{T^2} = \frac{\gamma M}{4\pi^2}, \quad T = 2\pi\sqrt{\frac{a^3}{\gamma M}}\right)$$

~~Времето е~~

$$\bar{g} = \frac{\gamma M}{(R+\frac{h}{2})^2} \approx \frac{\gamma M}{R^2}\left(1-\frac{h}{R}\right)$$

$$h = v_0 t_x - \frac{g t_x^2}{2} = \frac{g t_x^2}{2} \qquad (0 = v_0 - g t_x)$$

$$t_x = \sqrt{\frac{2h}{g}} \approx \sqrt{\frac{2hR^2}{\gamma M}\left(1+\frac{h}{R}\right)} \approx 105\,s$$



$$\cos\theta = \frac{R}{R+h}$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2},$$

$$\frac{R}{R+h} \approx 1 - \frac{h}{R}$$

$$\theta \approx \sqrt{\frac{2h}{R}} \approx 0{,}3\,rad$$

$$\frac{(R+h)^3}{P^2} = \frac{\gamma M}{4\pi^2}$$

$$P = 2\pi\sqrt{\frac{(R+h)^3}{\gamma M}} \approx 5{,}6\cdot 10^4\,s$$

$$t = \frac{\theta}{2\pi} P \approx 2800\,s$$

$$\Delta t = t - t_x \approx 2700\,s \approx 0{,}75\,h$$

$$\Delta t \approx 45\,min$$

Чернова

(1) $m_{min} = 16^m$ $m_{max} \simeq 6^m$

$m_{max} - m_{min} = -2,5 \lg \frac{L_{max}}{L_{min}}$

$\frac{L_{max}}{L_{min}} = 10^{0,4(m_{min} - m_{max})} = 10^{0,4 \cdot 10} = 10^4$

$L = 4 \pi R^2 \sigma T^4$, $T = const \Rightarrow L \sim R^2$

$\frac{R_{max}^2}{R_{min}^2} = 10^4$, $\frac{R_{max}}{R_{min}} = 10^2$

$t = \frac{P}{2} \simeq 200d$ $R_{max} - R_{min} = 99 R_{min} \simeq 10^2 R_{min}$

$\bar{v} = \frac{R_{max} - R_{min}}{t} \simeq 100 \frac{R_{min}}{t} = \frac{R_{max}}{t}$

Ако $R_{min} = 5 \cdot 10^2 R_\odot$:

$\bar{v} = 100 \cdot \frac{5 \cdot 10^2}{200} \simeq 2,5 \cdot 10^2 R_\odot / d$

$R_\odot \simeq 7 \cdot 10^5 km$

$1d = 24 \cdot 3600 s = 86400 s$

$3600 \cdot 24$, 14400, $+72$, 86400

$\bar{v} \simeq \frac{2,5 \cdot 10^2 \cdot 7 \cdot 10^5}{8,64 \cdot 10^4} = \frac{21}{8,64} \approx \frac{21}{8,6}$

$86 \cdot 2 = 172$ $210 : 86 \approx 2,4 \ldots$ -172, 380, -344

$86 \cdot 4 = 344$

$\bar{v} \simeq 2,4 \cdot 10^3 km/s$

Ако $R_{max} = 5 \cdot 10^2 R_\odot$

$\bar{v} = \frac{5 \cdot 10^2}{200} = 2,5 R_\odot / d \simeq 24 km/s$

По-вероятен е случаят $R_{max} = 5 \cdot 10^2 R_\odot$
($5 \cdot 10^4$ е твърде голям размер (R_{max}))

$\frac{8.10^{4^2}.7.10^5}{150.10^6} \approx 2,3.10^2 AU$ → твърде голям размер за звезда

(2) $\mu_{O_2} = 2.16 g/mol = 32 g/mol = 0,032 kg/mol$

~~$m = N\mu_{O_2} = 2,5.10^{29}.3,2.10^{-2} = 8.10^{27} kg$~~

(при $N = 2,5.10^{29}$)

$2,5.3,2$ = 50 + 75 = 8,00

$n = \frac{N}{N_A}$

$N_A \approx 6,2.10^{23} mol^{-1}$

$M = \rho.\frac{4}{3}\pi R^3$

$m = n\mu_{O_2} = \frac{N\mu_{O_2}}{N_A} = \frac{8.10^{27}}{6,2.10^{23}} \approx 1,3.10^4 kg$

~~$m \approx 13 kg$~~

$m \approx 1,3.10^4 kg \approx 13t$

~~$m = 2,5.10^{29}$~~

$\rho = 1,24.10^3 kg/m^3$ $R = 7,64.10^5 m$

$M = \rho.\frac{4}{3}\pi R^3 = 1,24.10^3.\frac{4}{3}.3,14.(7,64)^3.10^{15}$

~~7,64.7,64~~ 7,6.7,6 = 456 + 532 = 57,76 ≈ 57,8

~~57,8.~~ 58.7,6 = 348 + 406 = 440,8 ≈ 441

$(7,64)^3 \approx 440$

$M = 1,24.4.\frac{3,14}{3}.440.10^{18} = 2,2.10^{21} kg$

1,24.4 = 4,96 ≈ 5

440.5 = 2200

Височината на атмосферата (или поне частта с по-голямата част от молекулите) ще е в слой с $h \sim 10^0 \div 10^1 km \Rightarrow h \ll R$

$g \approx \frac{\gamma M}{R^2}$

$pS = mg$, $S = 4\pi R^2$

$p \cdot 4\pi R^2 = \frac{\gamma m}{R^2} \cdot \rho \cdot \frac{4\pi R^3}{3}$

$p = \frac{\rho \gamma m}{3R} = \frac{1,24\cdot10^3 \cdot 6,67\cdot10^{-11} \cdot 1,3\cdot10^4}{3 \cdot 7,64 \cdot 10^5}$

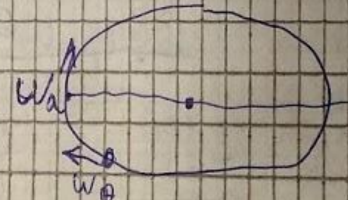
$1,3 \cdot 1,3$: 39 + 13 = 169

$6,7 \cdot 1,7$: 469 + 67 = 1139 $\approx 11,4$

$7,6 \cdot 3 = 22,8$

$p \approx \frac{11,4 \cdot 10^{-4}}{22,8 \cdot 10^5} \approx 5 \cdot 10^{-10}$ Pa

(3)



За 1г. апс. линия се отмества на

$\theta = \omega_a T_\oplus$

Земята изминава θ за

$t = \frac{\omega_a T_\oplus}{\omega_{\oplus p}}$

$\omega_{\oplus p} = \frac{1}{a(1-e)} \sqrt{\frac{\gamma M}{a}} \sqrt{\frac{1+e}{1-e}}$

$\sqrt{\frac{\gamma M}{a}} = \frac{2\pi}{T_\oplus} \sqrt{\frac{a^3}{T_\oplus^2}}$

$V_{\oplus p} = \sqrt{\frac{\gamma M}{a}} \sqrt{\frac{1+e}{1-e}}$ $r = a(1-e)$

$\omega_{\oplus p} = \frac{V_{\oplus p}}{r} = \sqrt{\frac{\gamma M}{a^3}} \sqrt{\frac{1+e}{(1-e)^3}}$

$\frac{a^3}{T_\oplus^2} = \frac{\gamma M}{4\pi^2} \Rightarrow \sqrt{\frac{\gamma M}{a^3}} = \frac{2\pi}{T_\oplus}$

$e = 0{,}0167 \ll 1$

$$\sqrt{\frac{1+e}{(1-e)^3}} = (1+e)^{\frac{1}{2}} \cdot (1-e)^{-\frac{3}{2}} \approx \left(1+\frac{e}{2}\right)\left(1+\frac{3e}{2}\right) =$$

$$= 1 + \frac{3}{2}e + \frac{e}{2} + \frac{3}{4}e^2 \approx 1+2e$$

$1{,}000 - 0{,}034 = 0{,}966$

$$\omega_{\oplus p} = \frac{2\pi}{T_\oplus}(1+2e)$$

$$\omega_a = \frac{2\pi}{T_a}$$

$$t = \frac{2\pi}{T_a} \cdot T_\oplus \cdot \frac{T_\oplus}{2\pi(1+2e)} = \frac{T_\oplus^2}{T_a(1+2e)}$$

$$t = \frac{T_\oplus^2}{T_a}(1-2e) \approx \frac{1^2 \cdot 0{,}97}{112 \cdot 10^3} \cdot 365{,}25\,d \cdot 24^h$$

$365{,}25 \cdot 24$: $1460 + 730 = 10760$; $+6 = 10766\,h$

$10766 \cdot 0{,}03 = 322{,}98 \approx 323$

$112 \cdot 9 = 1008$

$10766 - 323 = 10443$

$$t = \frac{10443}{112 \cdot 10^3}\,h$$

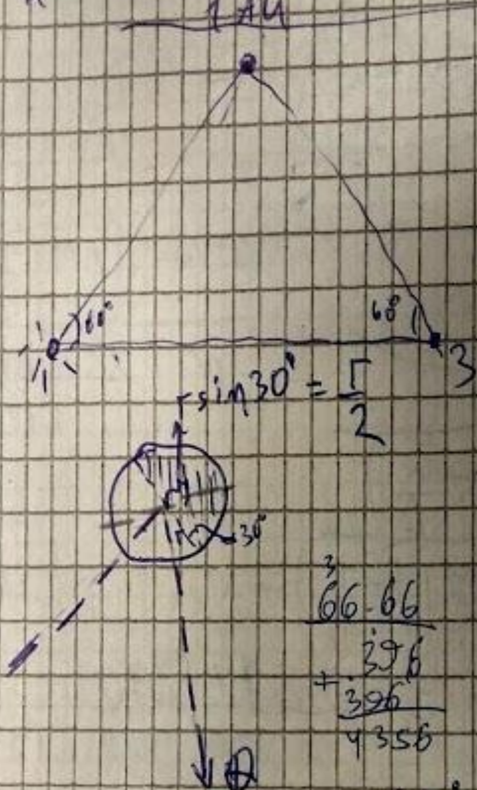
$112 \cdot 93$: $336 + 1008 = 10416$

$t \approx 93 \cdot 10^{-3}\,h = 0{,}093h$

$3 \cdot 24 + 11 - 4 = 72 + 7 = 79$

(4)

(5) $E_0 \sim 3\sqrt{r}^2$



134

57 - на границата

134
-57
77

134
+57
191

290 . 32
580
+87
9280

66 . 66
396
+396
4356

4356 . 3,1
4356
+13068
13503,6

49 . 49
441
+176
2201

2201 . 3,1
2201
+6603
6823,1

9280 . 2 = 4640
-8
12

(5)

$$V_0 + V_M = \sqrt{\gamma M\left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$r = R, \quad a = \frac{R+R+h}{2} = R + \frac{h}{2}$$

$$V_0 + V_M = \sqrt{\gamma M\left(\frac{2}{R} - \frac{2}{2R+h}\right)} = \sqrt{\gamma M\left(\frac{4R + 2h - 2R}{R(2R+h)}\right)} \approx$$

$$\approx \sqrt{\frac{\gamma M}{R}\left(1 + \frac{h}{R}\right)}$$

$$V_0 = \sqrt{\frac{\gamma M}{R}\left(1+\frac{h}{R}\right)} - V_M \approx 20 \text{ km/s}$$

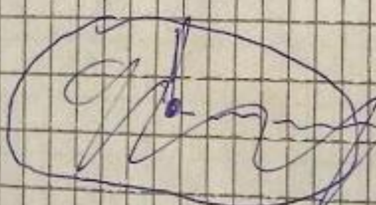
$$V_M = \frac{2\pi R_M}{T_M} = \frac{2\cdot 3{,}14\cdot 1700}{27{,}3\cdot 3600\cdot 24} \approx \frac{[illegible]}{27{,}3\cdot 24}$$

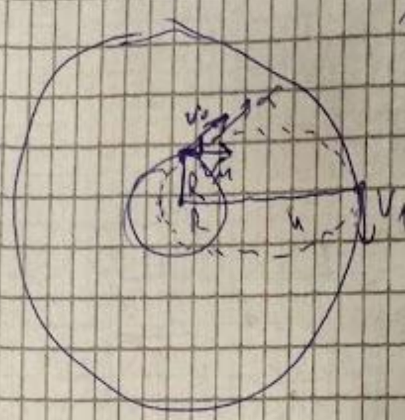
$$V_M \approx 0{,}025 \text{ km/s}$$

$$\sqrt{\frac{6{,}67\cdot 10^{-11}\cdot 6\cdot 10^{24}}{80\cdot 1{,}7\cdot 10^{6}}\left(1+\frac{70}{1700}\right)} \approx \sqrt{\frac{1{,}04\cdot 40\cdot 10^{13}\cdot 10^{[illegible]}}{136\cdot 10^{6}}} =$$

$$= 10^5\sqrt{\frac{4{,}16}{136}} = 10^5\cdot\frac{2{,}1}{11{,}5} \approx 2\cdot 10^4 \text{ m/s} = 20 \text{ km/s}$$

$$\frac{6{,}67\cdot 6}{40{,}2} \qquad \frac{1{,}7\cdot 80}{1360}$$





$$m v_0 \cos\alpha R = m v_1 (R+h)$$

$$\frac{m v_0^2}{2} - \frac{\gamma m M}{R} = \frac{m v_1^2}{2} - \frac{\gamma m M}{R+h}$$

$$\frac{v_0^2}{2} - \frac{\gamma M}{R} = \frac{v_0^2 \cos^2\alpha}{2}\frac{R^2}{(R+h)^2} - \frac{\gamma M}{R+h}$$

$$\frac{v_0^2}{2}\left(1 - \cos^2\alpha \frac{R^2}{(R+h)^2}\right) = \gamma M\left(\frac{1}{R} - \frac{1}{R+h}\right) = \frac{\gamma M h}{R^2}$$

$$v_0^2 = \frac{2\gamma M h}{R^2\left(1 - \cos^2\alpha \frac{R^2}{(R+h)^2}\right)}$$

$v_0 \to \min$ при $\cos^2\alpha = 0$, $\alpha = \frac{\pi}{2}$

$\Rightarrow$ вертик. наверх

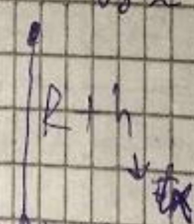
$$v_0 = \sqrt{\frac{2\gamma M h}{R^2}} = \sqrt{\frac{2 \cdot 6{,}67 \cdot 10^{-11} \cdot 6 \cdot 10^{24} \cdot 40}{80 \cdot (1{,}7)^2 \cdot 10^{12}}} =$$

$$= \frac{\sqrt{2\gamma M h}}{R} = \frac{\sqrt{2 \cdot 6{,}67 \cdot 10^{-11} \cdot 6 \cdot 10^{24} \cdot 40000}}{1{,}7 \cdot 10^6}$$

$$= \frac{1}{1{,}7 \cdot 10^6} \cdot \sqrt{\frac{40{,}2 \cdot 10^{17} \cdot 4}{4}} \approx \frac{2{,}8 \cdot 10^9}{1{,}7 \cdot 10^6} \approx 1{,}5 \cdot 10^3 \text{ m/s}$$

$$v_0 \approx 1{,}5 \text{ km/s}$$

2,3·2,3 / 4,69 / 46 / 5,29



$$a = \frac{R+h}{2}$$

$$\frac{a^3}{T^2} = \frac{\gamma M}{4\pi^2} \qquad \frac{(R+h)^3 \cdot 4}{8\, t_x^2} = \frac{\gamma M}{4\pi^2}$$

$$t_x^2 = \frac{2\pi^2 (R+h)^3}{\gamma M}$$

$$T = \sqrt{\frac{4\pi^2 a^3}{\gamma M}} = \sqrt{\frac{\pi^2 (R+h)^3}{2\gamma M}} = 2\pi\sqrt{\frac{(R+h)^3}{8\gamma M}} \approx$$

$$\approx R\sqrt{\frac{v_0^2 R}{2\gamma M}\left(1+\frac{3h}{R}\right)} \approx \frac{R\,5c}{v_0}\sqrt{\frac{R}{h}\left(1+\frac{3h}{R}\right)} =$$

$$v_0 = \sqrt{\frac{2\gamma M h}{R^2}} = \sqrt{\frac{2\gamma M}{R}}\sqrt{\frac{h}{R}}$$

$$1{,}7\cdot 1{,}7 \quad 119 \quad +17 \quad 289$$

$$= \frac{5c R}{v_0}\sqrt{\frac{R}{h}+3} = \frac{3\cdot 1{,}7\cdot 10^3}{1{,}5}\sqrt{\frac{1700}{70}+3} =$$

$$= \frac{6\cdot 10^3}{1{,}5}\cdot\sqrt{27{,}5} =$$

$$= 4\cdot 10^3\cdot 3\cdot 1{,}7 \approx 2\cdot 10^4\ s$$

$$1{,}7\cdot 12 \quad 34 \quad +17 \quad 20{,}4$$

$$\sqrt{\frac{2\cdot 7\cdot 10^{4}\cdot 2{,}9\cdot 10^{2}\ [illegible]}{40{,}2\cdot 10^{3}}} = \sqrt{\frac{42\cdot 10^3}{40{,}2}} \approx 105\ s$$

$$\Theta = \arccos\frac{R}{R+h}$$

$$\cos\Theta \approx 1-\frac{\Theta^2}{2}$$

$$\cos\Theta = \frac{R}{R+h} = \left(1+\frac{h}{R}\right)^{-1} \approx 1-\frac{h}{R}$$

$$\Theta \approx \sqrt{\frac{2h}{R}} \approx \sqrt{\frac{140}{1700}} \approx 0{,}1\sqrt{8} = 0{,}2\cdot 1{,}4 \approx 0{,}3\ rad$$

$$T = 2\pi\sqrt{\frac{(R+h)^3}{\gamma M}} = t_x\sqrt{8} \approx 2{,}8\,t_x = 5{,}6\cdot 10^4\ s$$

$$t_x + \Delta t = \frac{2\Theta}{2\pi}T = \left(1+\frac{\Theta}{\pi}\right)T$$

$$\Delta t = \frac{\Theta}{2\pi}T - t_x = \frac{0{,}3}{6{,}3}\cdot 5{,}6\cdot 10^4 - 2\cdot 10^4 + 5{,}6\cdot 10^4 =$$

$$\approx \left(\frac{5{,}6}{21} - 2 + 5{,}6\right)\cdot 10^4 \approx 3{,}3\cdot 10^4\ s$$

$$0{,}28\cdot 10^4 = 2800$$