Софо - 06 Задача 1

$$E(t) = E_{max} \cdot e^{-kt}$$

$E(0) = E_{max} \longrightarrow 15.05.1987$

$E(t_1) = E_{max} \cdot e^{-k \cdot t_1} \longrightarrow 04.02.1994$

$E(t_2) = E_{max} \cdot e^{-k \cdot t_2} \longrightarrow 21.04.1989$

$$\frac{E(t_1)}{E(t_2)} = 10^{0,4 \Delta m}$$

$$\frac{E_{max} \cdot e^{-k \cdot t_1}}{E_{max} \cdot e^{-k \cdot t_2}} = e^{\ln(10) \cdot 0,4 \Delta m}$$

$$e^{k\underbrace{(t_2 - t_1)}_{= \Delta t}} = e^{\ln(10) \cdot 0,4 \Delta m}$$

$$\Rightarrow \quad k \cdot \Delta t = \ln(10) \cdot 0,4 \Delta m$$

$$k = \frac{\ln(10) \cdot 0,4 \Delta m}{\Delta t} = \frac{2,2 \cdot 0,4 \cdot 5}{440} =$$

$$= \frac{4,4}{440} \approx \frac{1}{100}$$

$E \sim \frac{1}{D^2} \rightarrow$ диаметър на обектива

$$\Rightarrow \quad \frac{E_{око}(t_1)}{E_{обектив}(t_2)} = 10^{0,4 \Delta m}$$

$$\left(\frac{D_{обектив}}{D_{око}}\right)^2 = 10^{0,4 \Delta m}$$

$D_{око} = 0,6$ cm

$D_{обектив} = 6$ cm

$$\Rightarrow \quad \cancel{2} \lg\left(\frac{D_{обектив}}{D_{око}}\right) = \frac{\cancel{2}}{5} \Delta m$$

$$\Delta m = 5 \cdot \lg 10 = 5$$

$m_{гр., око} \approx 6^m$

$$\frac{E(t_1)}{E(0)} = 10^{0,4(m_{max} - m_{гр., око})}$$

$$e^{-k \cdot t_1} = e^{\ln(10) \cdot 0,4(m_{max} - m_{гр., око})}$$

$$m_{гр., око} - \frac{k \cdot t_1}{\ln(10)(0,4)} = m_{max}$$

$$6 - \frac{\frac{1}{\cancel{100}} \cdot t_1}{\cancel{88}} = m_{max} \quad \Rightarrow \quad m_{max} = 6 - \frac{t_1[d]}{88} \approx 6 - \frac{266}{88} \approx \boxed{3^m}$$

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Задача 2

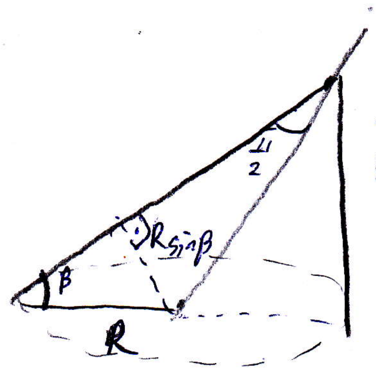
еклиптична ширина

θ - аберация $= \sin\beta \dfrac{V}{c}$

R - радиус на орбитата

r - разстояние до звездата

$\pi = 2\sin\beta \dfrac{R}{r}$



$\sin\frac{\pi}{2} = \dfrac{R\sin\beta}{r}$

$\frac{\pi}{2} \approx 0$

$\Rightarrow \pi = 2\dfrac{R\sin\beta}{r}$

$\pi = 5\theta$

$5 \cdot \cancel{\sin\beta} \cdot \dfrac{V}{c} = 2\cancel{\sin\beta}\dfrac{R}{r}$

$\dfrac{5}{2}\dfrac{r}{c} = \dfrac{R}{V} \quad |^2$

$V = \sqrt{\dfrac{\gamma M}{R}}$

$\left(\dfrac{5}{2}\dfrac{r}{c}\right)^2 = \dfrac{R^3}{\gamma M}$

$R = \sqrt[3]{\gamma M\left(\dfrac{5}{2}\dfrac{r}{c}\right)^2} = \sqrt[3]{6{,}67.10^{-11}.2.10^{30}.\left(\dfrac{5}{2}\dfrac{2{,}2.206265.150.10^{9}}{3.10^{8}}\right)^2} =$

$= \sqrt[3]{2{,}7.10^{20}.\left(5{,}1.10^{8}\right)^2} \approx \sqrt[3]{2{,}7.10^{20}.2{,}6.10^{17}} \approx \sqrt[3]{7{,}3.10^{37}} =$

$= \sqrt[3]{73.10^{36}} = 9.10^{12}.\sqrt[3]{10} \approx 20.10^{12} = 2.10^{13}\,m = \dfrac{4\cancel{2.10^{13}}10^2}{3\cancel{1{,}5.10^{11}}} =$

$\dfrac{400}{3} \approx \boxed{133\ au}$

Соф-06 Задача. 3

$\text{III}^{\text{та}}$ зако на Кеплер:

$$\frac{a_{мкс}^3}{P_{мкс}^2}=\frac{a_г^3}{P_г^2}$$

$a_г$ – голяма полуос на гантата

$P_г$ – орбитален период на гантата

$$\frac{a_{мкс}^3}{P_{мкс}^2}=\frac{(a_{мкс}-\Delta a)^3}{(P_{мкс}-\Delta P)^2}$$

$a_г = a_{мкс}-\Delta a$

$P_г = P_{мкс}-\Delta P$

$$\frac{\cancel{a_{мкс}^3}}{\cancel{P_{мкс}^2}}=\frac{\cancel{(a_{мкс})}^3}{\cancel{(P_{мкс})}^2}\cdot\frac{\left(1-\frac{\Delta a}{a_{мкс}}\right)^3}{\left(1-\frac{\Delta P}{P_{мкс}}\right)^2}$$

$$\boxed{V_{мкс}=\frac{2\pi a_{мкс}}{P_{мкс}}=\frac{2\pi\cdot 6600}{93}=\frac{2\cdot 3{,}14\cdot 6600}{93}=\approx 440\,\frac{km}{min}\approx 26\,400\,\frac{km}{h}}$$

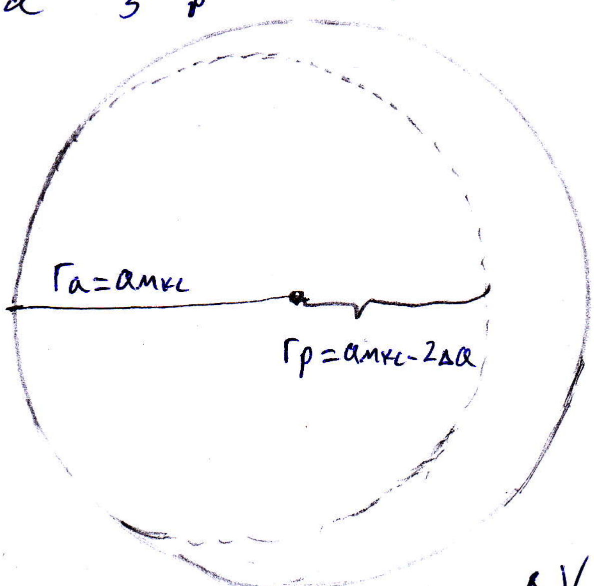
$$\left(1-\frac{\Delta P}{P_{мкс}}\right)^2=\left(1-\frac{\Delta a}{a_{мкс}}\right)^3,\quad \frac{\Delta P}{P_{мкс}} \text{ и } \frac{\Delta a}{a_{мкс}} \ll 0$$

$$\Rightarrow (1+x)^n \simeq 1+nx$$

$$\cancel{1}-\frac{2\Delta P}{P_{мкс}}=\cancel{1}-3\frac{\Delta a}{a_{мкс}}$$

$$\Rightarrow \Delta a=\frac{2}{3}\frac{a_{мкс}}{P_{мкс}}\Delta P$$

$$\Rightarrow \frac{\Delta a}{a}=\frac{2}{3}\frac{\Delta P}{P},\quad \frac{\Delta P}{P}\approx\frac{1}{30}$$



$V_{min}=\Delta V=V-V_a$

$$V_a=\sqrt{\frac{\gamma M}{a_г}}\sqrt{\frac{1-e}{1+e}}=$$

$$=\sqrt{\frac{\gamma M}{a-\Delta a}}\sqrt{\frac{r_p}{r_a}}=\sqrt{\frac{\gamma M}{a-\Delta a}}\sqrt{\frac{a-2\Delta a}{a}}=$$

$$=\sqrt{\frac{\gamma M\cdot a\left(1-\frac{2\Delta a}{a}\right)}{a^2\left(1-\frac{\Delta a}{a}\right)}}=\sqrt{\frac{\gamma M}{a}}\cdot\sqrt{\frac{1-\frac{2\Delta a}{a}}{1-\frac{\Delta a}{a}}}=$$

$$=V_{мкс}\cdot\sqrt{\frac{1-\frac{4}{3}\frac{\Delta P}{P}}{1-\frac{2}{3}\frac{\Delta P}{P}}}\simeq V_{мкс}\cdot\sqrt{\frac{43}{44}}\simeq 0{,}99\,V_{мкс}$$

$$\Delta V=V-V_a=0{,}01\,V_{мкс}\simeq$$

$$\simeq 264\,\frac{km}{h}$$

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Задача 4

Ω_{M51} – пространствен ъгъл M51

$\Omega_{NGC7000}$ – пространствен ъгъл на NGC7000

$m_{M51} = 8^m$

$m_{NGC7000} = 4^m$

$m_{0,M51}$ – повърхностна зв. величина на M51

$m_{0,NGC7000}$ – повърхностна зв. величина на NGC7000

$m_{гр}$ – гранична зв. величина (която е нужна, за да видим изображение)

$$\frac{E}{E_0} = 10^{0,4(m_0 - m)}$$

$$\frac{\Omega \cdot \cancel{E_0}}{\cancel{E_0}} = 10^{0,4(m_0 - m)}$$

$$m_0 = \frac{5}{2}\lg(\Omega) + m$$

$$\frac{E_0}{E_N} = 10^{0,4(m_{гр} - m_0)}$$

$$\frac{\cancel{E_0}}{\cancel{E_0} \cdot N} = 10^{0,4(m_{гр} - m_0)}$$

$$-\frac{5}{2}\lg(N) = m_{гр} - m_0$$

$$m_{гр} = m_0 - \frac{5}{2}\lg(N) = \frac{5}{2}\lg(\Omega) + m - \frac{5}{2}\lg(N)$$

$$\downarrow$$

$$m_{гр,M51} = m_{гр,NGC}$$

$$\Rightarrow \frac{5}{2}\lg(\Omega_{M51}) + m_{M51} - \frac{5}{2}\lg(N_{M51}) = \frac{5}{2}\lg(\Omega_{NGC7000}) + m_{NGC7000} - \frac{5}{2}\lg(N_{NGC7000})$$

$$\frac{5}{2}\lg(N_{NGC7000}) = m_{NGC7000} - m_{M51} + \frac{5}{2}\lg(N_{M51}) + \frac{5}{2}\lg\left(\frac{\Omega_{NGC7000}}{\Omega_{M51}}\right) \quad \Big| : \frac{5}{2}$$

$$\lg(N_{NGC7000}) = \frac{2}{5}(m_{NGC7000} - m_{51}) + \lg\left(N_{M51} \cdot \frac{\Omega_{NGC7000}}{\Omega_{M51}}\right)$$

$$N_{NGC7000} = 10^{0,4(m_{NGC7000} - m_{51})} \cdot N_{M51} \cdot \frac{\Omega_{NGC7000}}{\Omega_{M51}}$$

$$N_{NGC7000} = 10^{-1,6} \cdot 20 \cdot \frac{\overset{10}{\cancel{120}} \cdot 100}{\frac{\pi}{4} \cdot \cancel{11 \cdot 12}} = \frac{1}{\underset{2}{\cancel{40}}} \cdot \overset{1}{\cancel{80}} \cdot \frac{\overset{100}{\cancel{4000}}}{\cancel{40}} = \boxed{50}$$

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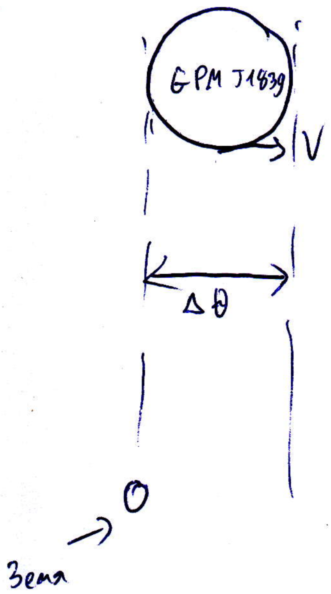
Задача. 5



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